An *ab initio* study of the ground and low-lying excited states of $LiBe^+$

Yang You, Chuan-Lu Yang*, Mei-Shan Wang, Xiao-Guang Ma, and Wen-Wang Liu

School of Physics and Optoelectronics Engineering, Ludong University, Yantai 264025, the People's Republic of China

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> **Abstract.** By using multi-reference configuration interaction method and large allelectron basis sets aug-cc-pwCV5Z, we have calculated the dense potential energy curves (PECs) of $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, $1^{1}\Pi$, and $1^{3}\Pi$ states of LiBe⁺ molecule. Based on the obtained PECs, the analytical potential energy functions (APEF) have been constructed with a Morse long-range potential function and nonlinear least squares method. The rotational and vibrational energy levels of the four states are determined by solving Schrödinger equation of nuclear movement with the APEFs. The spectroscopic parameters are deduced with the obtained rotational and vibrational energy levels.

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Key words: analytical potential energy function, excited state, spectroscopic parameters.

1 Introduction

Ultra-cold molecules have been brought in a great deal of interests in the past twenty years [1, 2], and now still arouse many researchers' interesting. Recently, some researchers have started to pay attention to the diatoms comprise of alkali and alkaline-earth metal elements. In 1993, using several theoretical methods and basis sets, Boldyrev *et al.* [1] studied the electronic structures of lithium containing diatomic molecules and ions. The equilibrium internuclear distance R_e and the harmonic vibrational frequency ω_e were obtained by using correlated second-order Moller-Plesset (MP2) with 6-311+G^{*} For the dissociation energy D_e , they used two methods, PMP4 and quadratic configuration interaction with single and double (triplet) (QCISD (T)) excitation, with the basis sets 6-311+G (2df). They found the QCISD (T) can give better results than PMP4 when compared with

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^{*}Corresponding author. *Email address:* ycl@ldu.edu.cn (C.-L. Yang)

the experimental values. Marino *et al.* [2] reported the potential energy curves (PEC) for the ground state and fourteen excited states of LiBe and LiBe⁺. Several spectroscopic parameters were also presented in their work. Safonov *et al.* [3] also performed a study on LiBe and LiBe+ by using self-consistent field method. The PECs and some spectroscopic parameters of $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, and $2^{1}\Sigma^{+}$ states of LiBe⁺ are also obtained. Using MRCI+Q and muti-reference averaged quadratic coupled-cluster (MRAQCC) methods including Davidson correction, Sun *et al.* [4] calculated the PECs for the X¹Σ⁺, $a^{3}\Sigma^{+}$, $A^{1}\Sigma^{+}$, $c^{3}\Sigma^{+}$, $C^{1}\Sigma^{+}$, $B^{1}\Pi$, and $b^{3}\Pi$ states. Their results showed that X¹Σ⁺, $a^{3}\Sigma^{+}$, $B^{1}\Pi$, and $b^{3}\Pi$ are weakly bound, while $A^{1}\Sigma^{+}$, $c^{3}\Sigma^{+}$, and $C^{1}\Sigma^{+}$ states are repulsive. The calculated spectroscopic parameters for the bound states with two methods show a little difference.

Beyond all question, these investigations should not be ignored and have made a significant effort for the later theoretical work. However, obvious differences occur in the reported spectroscopic parameters from the different author, especially for the D_e in the ground state. Besides, the analytical potential energy functions (APEF) of the PECs, which are very important for constructing an analytical function for the system containing more atoms or dynamical calculations such as photoassociation or photoassociation dissociation investigation, have not constructed. Therefore, the present paper focus on performing high accurate *ab initio* calculations for the PECs of LiBe⁺ and constructing the APEF for both the ground and the low-lying states.

2 Theoretical details

2.1 Computational details

The multi-reference configuration interaction (MRCI) [5, 6] method shows a good performance in dealing with the electron correlation. Combined with suitable basis sets and active space, MRCI can give the accurate PEC for molecule. The MRCI calculations are based on the optimized orbitals with the complete-active-space-self-consistent field (CASSCF) method. In the present work, the all-electron basis sets aug-cc-pwCV5Z [7] are used, which implies that (9s,4p,1d) [3s,2p,1d] for both Li and Be atoms. Orbitals and occupation schemes are referring to the irreducible representation (A₁/B₁/B₂/A₂) of C_{2v} group. All calculations are performed in the C_{2v} subgroup of the C_{∞v} point group because of the limit of the program. No core orbitals is used to include the correlation effect of the inner 1s electrons. That is to say, the orbitals of 2p of Li and Be are included in the active space for the excited states, which mean the active space consisted of 2 closedshell orbitals (2,0,0,0) and 8 active orbitals (4,2,2,0). Hence there are 2 active electrons in 8 orbitals in the correlation energy calculations. The PECs spanning a range from 1.50 to 21.45 Å with a step of 0.05 Å are used for both the ground and the low-lying excited states. All calculations are carried out with MOLPRO package [8].

2.2 Expression of APEF

The Morse long-range (MLR) potential function suggested by Le Roy [9, 10] is adopted to fit the PECs of LiBe⁺ because it can accurately reproduce the PECs in both the long and short separate distances. The general MLR function form is

$$V_{MLR}(R) = D_e \left\{ 1 - \frac{u_{LR}(R)}{u_{LR}(R_e)} \exp[-\phi_{MLR}(R)Y_p(R)] \right\},$$
(1)

Where the *R* and R_e are the interatomic distance and the equilibrium bond length, respectively. The corrected long-range (attractive) interaction energy is defined as

$$u_{LR}(R) = \sum_{i=1}^{last} D_{mi}(R) \frac{C_{mi}}{R^{mi}},$$
(2)

and

$$Y_p(R) = (R^p - R_e^p) / (R^p + R_e^p),$$
(3)

where $D_{mi}(R)$ is the damping function, which has been fairly widely used in recent years and especially proposed by Douketis *et al.* [11] The generalized damping function is

$$D_m^{DS(s)}(R) = \left\{ 1 - \exp\left[-\frac{b^{ds}(s)\rho R}{m} - \frac{c^{ds}(s)(\rho R)^2}{m^{1/2}} \right] \right\}^{m+s}.$$
 (4)

The parameters b(s) and c(s) are system-independent constants determined by optimizing the agreement with the *ab initio* (m = 6, 8, 10) damping function behavior that was determined by Kreek and Meath [12]. Here, we used the standard, as recommended by Douketis *et al.* [11] In addition, the system-dependent range parameter ρ for interacting atoms *A* and *B* is presented as

$$\rho \equiv \rho_{AB} = \frac{2\rho_A \rho_E}{\rho_A + \rho_E},\tag{5}$$

in which

$$\rho_A = \left(\frac{I_{\rho}^A}{I_{\rho}^H}\right)^{2/3},\tag{6}$$

and

$$\rho_B = \left(\frac{I_{\rho}^B}{I_{\rho}^H}\right)^{2/3}.$$
(7)

Here, ρ is defined in terms of the ratio of the ionization potential of the atom in question (I_{ρ}^{A}) to that of the H atom (I_{ρ}^{H}) . Finally, the function $_{uLR}(\mathbb{R})$ defines the long-range behavior of the potential energy function

$$V(R) \approx D_e - u_{LR}(R) = D_e - \sum_{i=1}^{last} D_{m_i}(R) \frac{C_{m_i}}{R_{m_i}},$$
(8)

where m_i (6, 8, and 10) are the dispersion coefficients. The exponent coefficient function is

$$\phi_{MLR}(R) = [1 - Y_p(R)] \sum_{i=1}^{17} \phi_i Y_p(R)^i + Y_p(R) \phi_{\infty}, \qquad (9)$$

and

$$\phi_{\infty} = In \left\{ \frac{2D_e}{u_{LR}(R)} \right\}. \tag{10}$$

To minimize the possibility of long-range irregular potential function behavior, the power *p* in the definition of $Y_p(R)$ is chosen to be a small positive integer. Meanwhile, if the potential is to achieve the long-range behavior of Eq. (1), then $p > (m_{last} - m_1)$. In the entire function, $\phi_{MLR}(R)$ contains only a series of ϕ_i coefficients obtained by fitting. In addition, to estimate quantitatively the quality of the fitting process, the root mean square error (RMS) is also given by

$$RMS = \frac{1}{N} \sqrt{\sum_{i=1}^{N} (V_{APEF} - V_{ab \ initio})^2},\tag{11}$$

where V_{APEF} and $V_{ab initio}$ are the energies given by the fitted and *ab initio* calculations, respectively, and *N* is the number of points.

2.3 Calculations of ro-vibrational energy levels and spectroscopic parameters

Based on the obtained APEFs, the ro-vibrational energy levels of each state are obtained by solving the Schrödinger equation of nuclear movement using LEVEL8.0 [13] program package, and then the spectroscopic parameters are fitted with the Dunham series expansion [14]. According to the Dunham expression, the ro-vibrational energy T(v,N) of a given level is represented by

$$T(v,N) = \sum_{m,n} Y_{m,n} (v+1/2)^m [N(N+1)]^n,$$
(12)

where $Y_{m,n}$ are the Dunham coefficients. They are determined with the calculated energy levels using a least squares fitting procedure. The spectroscopic parameters are related to the coefficients as follows:

$$Y_{10} \approx \omega_e, Y_{20} \approx -\omega_e X_e, Y_{30} \approx \omega_e y_e$$

$$Y_{40} \approx \omega_e Z_e, Y_{01} \approx B_e, Y_{11} \approx -\alpha_e$$

$$Y_{21} \approx \gamma_e, Y_{02} \approx D_{rot}, Y_{12} \approx -\beta_e.$$
(13)

Thus, the spectroscopic parameters can be obtained. The R_e and dissociation energy D_e are obtained with the fitted APEFs.

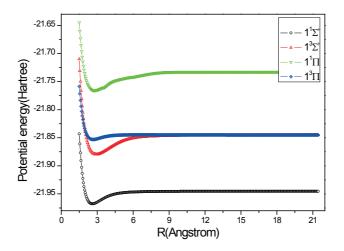


Figure 1: PECs for the $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, $1^{1}\Pi$, and $1^{3}\Pi$ states of LiBe⁺.

3 Results and discussion

3.1 PECs and APEFs

The *ab initio* PECs for the $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, $1^{1}\Pi$, and $1^{3}\Pi$ states of LiBe⁺ are depicted in Fig. 1. From the figure, it is obvious to see that the $1^{1}\Sigma^{+}$ state is the lowest one, which means that it is the ground state. Although the higher $1^{1}\Pi$ state demonstrates unsmooth to some extent, we can still obtain the APEF with an acceptable error. It may be improved with a larger active space.

As is known to all, the PECs in numerical form is not convenient for further applications, especially for dynamical calculation or construction of analytical potential energy surfaces containing many atoms. Therefore, we fitted the PECs into APEFs with the MLR function and non-linear squares fitting method. All parameters including ϕ_1 to ϕ_{17} , D_e , R_{ef} , R_e , the long-range coefficients C_6 , C_8 , C_{10} , and RMS are provided in Table 1. The smallest RMS is 0.427 cm⁻¹ for the $1^{1}\Sigma^{+}$ state shows the suitability of the MLR function for these states and the high quality of the fitting process. Because of the unsmooth PEC of the $1^{1}\Pi$ state, it has the largest RMS of 11.600 cm⁻¹, however, which is acceptable because it is still far less than the chemical accuracy 349.755 cm⁻¹.

3.2 Ro-vibrational energy levels and spectroscopic parameters

In order to calculate the vibrational and rotational energy levels for LiBe⁺, we use LEVEL8.0 [13] program package to solve the Schrödinger equation of the nuclear motion based on the obtained APEFs. We have found 31, 49, 53, and 12 levels for the $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, $1^{1}\Pi$, and $1^{3}\Pi$ states, respectively. For the sake of brevity, we only list the first 12 levels in the Table 2. They can be used as a helpful reference for further investigation because there is no theoretical or experimental data for these states in the literature.

	$1^{1}\Sigma^{+}$	$1^{3}\Sigma^{+}$	$1^{1}\Pi$	$1^3\Pi$
ϕ_1	-2.114964383761	-2.101630782656	-3.273987893657	-3.289110244343
ϕ_2	-4.56015005360010^{-1}	-5.880960498423	$4.296882414934 \times 10^{-1}$	5.774979399070
ϕ_3	$-8.479874817989{\times}10^{-1}$	$\text{-}1.010893013755\!\times\!10^{1}$	6.781378841748×10 ⁻²	$1.000226737263 \times 10^{1}$
ϕ_4	$-7.052308767331\!\times\!10^{-1}$	$\text{-}1.245785921524 \!\times\! 10^1$	5.649769650444	$1.078073906733\!\times\!10^{1}$
ϕ_5	-4.670449284978	$\textbf{-2.501588513311}{\times}10^{1}$	$\text{-}4.800171313213 {\times} 10^1$	1.584471978961
ϕ_6	$-4.311598391878\!\times\!10^{1}$	$\textbf{-6.334568612041}{\times}10^{1}$	$-1.012190695041\!\times\!10^2$	$1.219681953033\!\times\!10^{1}$
ϕ_7	$-6.798963326036 \times 10^{1}$	$1.399125448361 \times 10^{-1}$	$2.653529989442 \times 10^{2}$	$1.706804027643 \times 10^{2}$
ϕ_8	$1.942297391442 \times 10^{2}$	$2.609553787274 \times 10^{2}$	$1.648954205712 \times 10^{2}$	$1.431759261419 \times 10^{2}$
φ9	$5.537005603724 \times 10^{2}$	$6.822911767751 \times 10^{1}$	$-8.060433587352\!\times\!10^2$	$-9.349212834273 \times 10^{2}$
ϕ_{10}	$-2.831273187700 \times 10^{2}$	$-9.861895440541\!\times\!10^2$	$9.504936600774 \times 10^{2}$	$-1.198579965498 \times 10^{3}$
ϕ_{11}	$-1.928044560659 \times 10^3$	$-9.831775105220 \times 10^{2}$	$1.673560238893 \times 10^{3}$	$2.728385588932 \times 10^{3}$
ϕ_{12}	$-8.407552217229 \times 10^{2}$	$1.158949174812 \times 10^{3}$	$-4.548332134539 \times 10^{3}$	$4.691667396637 \times 10^3$
ϕ_{13}	$2.604063907177 \times 10^3$	$2.083937094551 \times 10^3$	$-3.415136955694 \times 10^{3}$	$-2.933134355532 \times 10^{3}$
ϕ_{14}	$2.585422462587 \times 10^{3}$	$3.389866159192 \times 10^{1}$	$6.981119098006 \times 10^{3}$	$-7.927725519817 \times 10^{3}$
ϕ_{15}	$-8.271197969562 \times 10^{2}$	$-1.561103837762 \times 10^{3}$	$4.840315158740 \times 10^{3}$	$-5.224890144534 \times 10^{2}$
ϕ_{16}	$-1.959558050074 \times 10^{3}$	$-1.006124359730 \times 10^{3}$	$-3.654437025084 \times 10^{3}$	$5.074112215088 \times 10^3$
ϕ_{17}	$-6.770827853067 \times 10^{2}$	$-1.911481397861\!\times\!10^2$	$-2.715033360966 \times 10^{3}$	$2.290940264178 \times 10^{3}$
D_e	$4.903607989794 \times 10^{3}$	$7.527014468295 \times 10^{3}$	$7.224677680126 \times 10^3$	$1.754200282361 \times 10^{3}$
R_{ef}	5.447656520127	7.144759770531	4.719602222884	8.488570525563
	2.599829038210	2.935215460676	2.784947543327	2.661490398269
C_6	$1.048646823287 \times 10^7$	$4.089246441117 \times 10^{6}$	$\textbf{-2.000140733514}{\times}10^8$	$-2.585549529673 \times 10^{9}$
C_8	$3.333820962753 \times 10^8$	$1.593938031969 \times 10^{8}$	$1.648158213712 \times 10^{10}$	$4.764999401183\!\times\!10^7$
C_{10}	$3.963128665362 \times 10^8$	$1.704954876670 \times 10^{9}$	$5.333071974397{\times}10^{10}$	$1.710070420243\!\times\!10^{12}$
RMS	4.27×10^{-1}	7.43×10^{-1}	1.16×10^{1}	7.20×10^{-1}

Table 1: The parameters of MLR for LiBe⁺. (D_e and RMS are in cm⁻¹, and R_e in Å.)

Based on the obtained ro-vibrational levels of those states, we have determined the spectroscopic parameters by using Dunham expansion of Eq. (12) and nonlinear least squares fitting method. The calculated spectroscopic parameters with the previously reported values [1-4] are tabulated in Table 3. It should be noticed that the values of D_e and R_e are fitted from the PECs with the MLR function.

For the ground state $1^{1}\Sigma^{+}$, our value of R_{e} is 2.600 Å, which is close to 2.629 Å of Boldyrev *et al.* [1], 2.635 Å of Marino *et al.*, [2] and 2.634 Å of Sun *et al.* [4] but smaller than 2.66 Å of Safonov *et al.* [3], respectively. The present dissociation energy D_{e} of 4903.6 cm⁻¹ is different from all the previously reported values. For examples, it is 286.8 cm⁻¹ larger than that of Boldyrev *et al.* [1], 391.9 cm⁻¹ larger than that of Marino *et al.* [2], 298.6 cm⁻¹ larger than that of Sun *et al.* [4], and 97.0 cm⁻¹ smaller than Safonov *et al.* [3]. It is shown that the calculational method and basis sets play more obvious effect on the D_{e} than that of R_{e} . The full CI with the Gaussian expansions of Slater-type orbitals (STO-6G) by Marino *et al.* [2] gives smaller D_{e} than that of QCISD(t)/6-311+G^{*} by Boldyrev *et al.* [1]. The present large basis sets aug-cc-pwCV5Z is responsible for the larger D_{e} . As

\mathcal{U}	$1^{1}\Sigma^{+}$	$1^{3}\Sigma^{+}$	$1^{1}\Pi$	1 ³ ∏
0	160.2783	136.4554	141.8240	121.3875
1	473.2341	405.8650	418.9235	354.4488
2	776.1739	670.8786	677.6471	573.1145
3	1069.1745	931.5224	911.0826	776.2302
4	1352.2139	1187.8030	1126.1489	963.0179
5	1625.1970	1439.7132	1339.6056	1133.1242
6	1887.9806	1687.2373	1558.5264	1286.5425
7	2140.3941	1930.3548	1782.8431	1423.3456
8	2382.2513	2169.0437	2011.2360	1543.265
9	2613.3529	2403.2829	2242.1027	1645.288
10	2833.4842	2633.0523	2473.7126	1727.5563
11	3042.4119	2858.3324	2704.2081	1787.975

Table 2: The vibrational levels (N = 0) for NaBe. (in cm⁻¹).

for the largest D_e given by Safonov *et al.*, [3] may be less reliable because the calculation was based on the SCF method. Our ω_e is in good agreement with those from all the four reports [1-4]. Our anharmonic frequency $\omega_e x_e$ is close to that of Boldyrev *et al.* [1], but both smaller than that of Safonov *et al.* [3] and Sun *et al.* [4] For the excited states $(1^3\Sigma^+ \text{ and } 1^3\Pi)$, the differences between our results and the reports in the literature [1-4] show a similar tendency. The reader can easily find the details in Table 3. Therefore, we omit the repetitive description. Unfortunately, no experimental data can be found for all the considered states. The present results can be used as a helpful reference for further investigation because they are performed with the high theoretical level of and the large all-electron basis sets, and are in agreement with the available theoretical reports in the literature.

4 Conclusions

The *ab initio* PECs of the ground and three low-lying excited states of LiBe⁺ are obtained using the high accurate CASSCF/MRCI method with large all-electron basis sets aug-ccpwCV5Z. By using the MLR function and the nonlinear least squares method, we deduce the APEFs for each state. The ro-vibrational levels and the spectroscopic parameters of the states are determined by solving the Schrödinger equation of nuclear movement based on the obtained APEFs. It is found that all the theoretical method, the basis sets and the active space impact on the spectroscopic properties. The obtained APEFs for the ${}^{1,3}\Sigma^+$ and ${}^{3}\Pi$ states with very little RMS should be reliable for further investigation, while that of ${}^{1}\Pi$ state need improve because of the relative larger RMS. These results could be helpful in the dynamical investigation of photoassciation or light dissociation of LiBe⁺ molecule.

	$1^{1}\Sigma^{+}$	$1^{3}\Sigma^{+}$	1 ¹ Π	1 ³ П
R_e	2.600	2.935	2.785	2.661
	2.629 ^{<i>a</i>}	2.953^{b}	2.839 ^d	2.694^{b}
	2.635 ^b	2.97 ^c		2.701 ^{<i>d</i>}
	2.66 ^c	2.966 ^d		
	2.634^{d}			
D_e	4903.6	7527.0	7224.7	1754.2
C C	4616.8 ^{<i>a</i>}	7339.6 ^b	3395.6 ^d	1532.5 ^b
	4511.7 ^b	7420.3 ^c		6976.7 ^d
	5000.6 ^c	1669.6^{d}		
	4605.0^d	100710		
ω_e	318.4	274.2	281.1	244.6
	320 ^{<i>a</i>}	274^{b}	256.3^d	235 ^b
	311 ^{<i>b</i>}	274 ^{<i>c</i>}		236.5^{d}
	320 ^c	270.8^{d}		
	315.3 ^d			
$\omega_{\rho} X_{\rho}$	4.310	2.290	1.572×101	5.501
	4.8^{b}	2.2^{b}	4.696^{d}	7.0^{b}
	8.2 ^{<i>c</i>}	5.4^{c}		6.954d
	4.762^{d}	2.173d		
$\omega_e y_e$	-2.409×10-2	7.748×10-3	1.872	4.845×10-1
	-5.241×10^{-2d}	-0.074×10^{-2d}	-2.703×10^{-2d}	-3.025×10^{-2d}
$\omega_e Z_e$	-1.855×10^{-3}	-2.379×10^{-4}	-9.333×10^{-2}	4.547×10^{-2}
	2.529×10^{-3d}	0.211×10^{-3d}	-0.468×10^{-3d}	-2.020×10^{-3d}
B _e	6.173×10^{-1}	4.949×10^{-1}	5.329×10^{-1}	5.984×10^{-1}
	0.616 ^b	0.490^{b}	0.533^{d}	0.590^{b}
	0.601 ^c	0.482^{c}		0.589d
	0.620^{d}	0.489d		
D _{rot}	-4.869×10^{-6}	-5.747×10^{-6}	-6.846×10^{-6}	-1.249×10^{-5}
	9.565×10^{-6d}	-6.539×10^{-6d}	0.923×10^{-6d}	14.61×10^{-6d}
α _e	9.179×10^{-3}	6.894×10^{-3}	4.875×10^{-3}	1.402×10^{-2}
	0.013^{b}	0.006^{b}	$1.232 \times 10^{-2} d$	0.021^{b}
	1.83×10^{-2c}	1.19×10^{-2c}		2.048×10^{-2d}
	$1.261{\times}10^{-2d}$	0.664×10^{-2d}		
γ_e	-3.234×10^{-4}	5.267×10^{-5}	-1.549×10^{-4}	
	-2.09310^{-4d}	$0.051{\times}10^{-4d}$	-1.631×10^{-4d}	-3.415×10^{-4d}
β_e	9.066×10^{-7}	3.737×10^{-8}	2.258×10^{-7}	2.739×10^{-6}
	7.017×10^{-8d}	6.856×10^{-8d}	-15.94×10^{-8d}	-50.47×10^{-8d}

Table 3: The spectroscopic parameters (in cm⁻¹ except R_e in Å) for the $1^{1}\Sigma^{+}$, $1^{3}\Sigma^{+}$, $1^{1}\Pi$, and $1^{3}\Pi$ states of LiBe⁺.

^{*a*} Data from Ref. 1. b Data from Ref. 2. c Data from Ref. 3. d Data from Ref. 4.

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References

- [1] A. I. Boldyrev, J. Simons, and P. V. R. Schleyer, J. Chem. Phys. 99 (1993) 8793-8804.
- [2] M. M. Marino, W. C. Ermler, C. W. Kern, and V. E. Bondybey, J. Chem. Phys. 96 (1992) 3756-3766.
- [3] A. A. Safonov, V. F. Khrustov, and N. F. Stepanov, J. Struct. Chem. 24.2 (1983) 321-323.
- [4] B. G. Sun, H. J. Chen, F. H. Liu, and Y. H. Yang, Acta Chimica Sinica. 69 (2011) 761-766.
- [5] H. J. Werner, and P. J. Knowles, J. Chem. Phys. 89 (1988) 5803-5814.
- [6] P. J. Knowles, H. J. Werner, Chem. Phys. Let. 145 (1988) 514-522.
- [7] B. P. Prascher, D. E. Woon, K. A. Peterson, T. H. Dunning Jr, and A. K. Wilson, Theo. Chem. Acc. 128 (2011) 69-82.
- [8] H. J. Werner, P. J. Knowles, R. Lindh, F. R. Manby and M. Schütz *et al.*, MOLPRO, is a package of *ab initio* programs, version 2010.1, see http://www.molpro.net.
- [9] R. J. Le Roy, Y. Huang, and C. Jary, J. Chem. Phys. 125 (2006) 164310.
- [10] R. J. Le Roy and R. D. E. Henderson, Mol. Phys. 105 (2007) 663-677.
- [11] C. Douketis, G. Scoles, S. Marchetti, M. Zen, and A.J. Thakkar, J. Chem. Phys. 76 (1982) 3057.
- [12] H. Kreek and W.J. Meath, J. Chem. Phys. 50 (1969) 2289.
- [13] R. J. Le Roy, LEVEL 8.0, A computer program for solving the radial Schrödinger equation for bound and quasibound levels, CPRR-661, University of Waterloo, 2007.
- [14] J. L. Dunham. Phys. Rev. 41 (1932) 721-724.