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NUMERICAL APPROXIMATION OF TRANSCRITICAL SIMPLE BIFURCATION POINT OF THE NAVIER-STOKES EQUATIONS*1)

He-yuan Wang
(*Department of Mathematics and Physics, Liaoning Institute of Technology, Jinzhou 121001, China*)
(*College of Sciences, Xi'an Jiaotong University, Xi'an 710049, China*)

Kai-tai Li
(*College of Sciences, Xi'an Jiaotong University, Xi'an 710049, China*)

Abstract

The extended system of nondegenerate simple bifurcation point of the Navier-Stokes equations is constructed in this paper, due to its derivative has a block lower triangular form, we design a Newton-like method, using the extended system and splitting iterative technique to compute transcritical nondegenerate simple bifurcation point, we not only reduces computational complexity, but also obtain quadratic convergence of algorithm.

Mathematics subject classification: 35A40, 65M60, 65N30, 65J15, 47H15.
Key words: Nondegenerate simple bifurcation point, Splitting iterative method, The extended system.

0. Introduction

Bifurcation problem of the Navier-Stokes equations has been studied rather extensively in the last years, see Li/Mei/Zhang(1986)[5], and M.Golubitsky/D.G.Schaefer(1988)[6], Allgower/E.Bohmer(1990) [7]. in this paper we discussed numerical approximate method of nondegenerate simple bifurcation point of the Navier-Stokes equations, the content of the paper is arranged as follows, first we introduce the Navier-Stokes equations and its operator form in the section 1, and discuss property of nondegenerate simple bifurcation points. in the section 2 we will construct a extended system as a tool for computing nondegenerate simple bifurcation points. in the section 3 we give a Newton-like method for computing transcritical nondegenerate simple bifurcation point, splitting iterative technique is used to compute transcritical nondegenerate simple bifurcation point of the Navier-Stokes equations. in the section 4 we will make numerical experiment.

1. Navier-Stokes Equation and its Nondegenerate Simple Bifurcation Point

We consider the stationary Navier-Stokes equations which has homogeneous boundary conditions

$$\begin{cases} -\nu\Delta u + (u\cdot\nabla)u + \nabla p = f, & x\in\Omega; \\ \mathrm{div} u = 0, & x\in\Omega; \\ u|_{\partial\Omega} = 0. \end{cases} \tag{1.1}$$

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Ω is a bounded and smooth domain of $R^m, m = 2, 3$, moreover $f \in [L^2(\Omega)]^m$, ν is the coefficient of kinematic viscosity.

It is well know that the uniqueness of solution of the stationary Navier-Stokes equations has only been proved under the assumptions that Reynolds number is sufficiently small, or f is sufficiently small, otherwise its solution may be not unique[1−3], for this reason it is very important to discuss efficient numerical algorithm of singular solution for Navier-Stokes equations.

Define function space

$$V = \{u \in [H_0^1(\Omega)]^m; \quad \text{div} u = 0\}$$
$$\mathcal{H} = \{u \in [L^2(\Omega)]^m; \quad \text{div} u = 0, u \cdot n|_{\partial\Omega} = 0\},$$

n denotes the outward normal vector on $\partial\Omega$. the scalar product and norm of $L^2(\Omega)^m$ are denoted by $(\cdot, \cdot)$, $|\cdot|$ on $\mathcal{H}$, Define the following scalar product on V

$$((u, v)) = (\nabla u, \nabla v), \quad \forall u, v \in V$$

$||\cdot||$ denotes its corresponding norm, variational formulation of the Navier-Stokers equations may be stated as follows [1][2]

$$\lambda a_0(u, v) + a(u, u, v) - (f, v) = 0, \quad \forall v \in V, \tag{1.2}$$

where $\lambda = \nu = Re^{-1}$ bilinear from $a_0(\cdot, \cdot)$ and trilinear form $a(\cdot, \cdot, \cdot)$ are defined by

$$a_0(u, v) = (\nabla u, \nabla v), \quad \forall u, v \in V,$$
$$a(u, v, w) = \int_\Omega (u \cdot \nabla) v \cdot w dx, \quad \forall u, v, w \in V.$$

introduce bilinear from $B(\cdot, \cdot) : V \times V \to V'$

$$< B(u, v), w >= a(u, v, w), \quad \forall u, v, w \in V,$$

where $< \cdot, \cdot >$ denotes duality pairing on $V' \times V$. let $T(u) = \mathcal{A}^{-1}[B(u, u) - f]$, where $\mathcal{A}$ is stokes operator , then operator form of the Navier-Stokes equations can be writ as follows[1][2]

$$G(u, \lambda) := \lambda u + T(u) \tag{1.3}$$

it is Frechet differentiable and $D_u G(u, \lambda) = \lambda I + T'(u)$, it is clear that $\forall u \in V, T'(u)$ is a compact operator form V into V [1][2] , and $G : V \times R \to V$ is a nonlinear Fredholm operator with 0-index,

In the sequel the subindex 0 indicate the evaluations of function at the point (u_0, λ_0). with some calculation, we obtain:

$$D_u G_0 = \lambda_0 I + T'(u_0) = \lambda_0 I + \mathcal{A}^{-1}[B(u_0, \cdot) + B(\cdot, u_0)], \tag{1.4}$$
$$D_u G_0^* = \lambda_0 I + T'^*(u_0) = \lambda_0 I + \mathcal{A}^{-1}[B^*(u_0, \cdot) + B^*(\cdot, u_0)], \tag{1.5}$$
$$D_{uu} G_0 = T''(u_0) = \mathcal{A}^{-1}[B(\cdot, \cdot) + B(\cdot, \cdot)], \tag{1.6}$$
$$D_\lambda G_0 = u_0; \quad D_{u\lambda} G_0 = I; \quad D_{\lambda\lambda} G_0 = 0, \tag{1.7}$$

Setting ϕ, ψ are eigenfunction of $D_u G_0$ and $D_u G_0^*$ corresponding to 0 eigenvalue respectively, namely

$$Ker(D_u G_0) = Span\{\phi\}, \quad ||\phi|| = 1 \tag{1.8}$$
$$Ker(D_u G_0^*) = Span\{\psi\}, \quad ||\psi|| = 1 \tag{1.9}$$
$$((\phi, \psi)) = 1 \tag{1.10}$$

Fredholm theory shows that

$$\text{Range}(D_u G_0) = \{u \in V, ((u, \psi)) = 0\} \tag{1.11}$$

$$\mathrm{Range}(D_uG_0^*) = \{u \in V, ((u,\phi)) = 0\} \tag{1.12}$$

moreover

$$V = Ker(D_uG_0) \oplus Range(D_uG_0^*) = Ker(D_uG_0^*) \oplus Range(D_uG_0) \tag{1.13}$$

Assume that point(u_0,λ_0) satisfying

(H_1) $G_0 = G(u_0,\lambda_0) = 0$

(H_2) D_uG_0 is a Fredholm operator, 0 is a eigenvalue of D_uG_0 with simple algebraic multiplicity

(H_3) $D_\lambda G_0 \in \mathrm{Rang}(D_uG_0)$, namely there exists $v_\lambda \in Range(D_uG_0^*)$, such that

$$D_uG_0v_\lambda + D_\lambda G_0 = 0 \tag{1.14}$$

we say that (u_0,λ_0) is simple bifurcation point of the Navier-Stokes equations (1.3)[2]. let

$$\alpha = ((\psi, D_{uu}G_0\phi\phi)) = 2a(\phi,\phi,\psi)$$

$$\begin{aligned}\beta &= ((\psi, D_{uu}G_0v_\lambda\phi + D_{u\lambda}G_0\phi)) = ((\psi, D_uDG_0(v_\lambda,1)\phi))\\ &= a(v_\lambda,\phi,\psi) + a(\phi,v_\lambda,\psi) + a_0(\phi,\psi)\\ \gamma &= ((\psi, D_{uu}G_0v_\lambda v_\lambda)) + 2((\psi, D_{u\lambda}G_0v_\lambda)) + ((\psi, D_{\lambda\lambda}G_0))\\ &= ((\psi, D^2G_0(v_\lambda,1)^2)) = 2a(v_\lambda,v_\lambda,\psi) + 2a_0(v_\lambda,\psi)\end{aligned}$$

if

$$d = \beta^2 - \alpha\gamma > 0,$$

we say that (u_0,λ_0) is nondegenerate simple bifurcation point of the Navier-Stokes equations (1.3), further if $\alpha \neq 0$, (u_0,λ_0) is transcritical nondegenerate simple bifurcation point[4].

2. The Extended System of Nondegenate Simple Bifurcation Point

In the neighborhood of bifurcation point (u_0,λ_0), classical numerical method, for example, Newton method is invalid, thereafter, we will construct a extended system, which make bifurcation point (u_0,λ_0) of (1.3) into nonsingular solution of the extended system. according to reference [8], we set up the following extended system:

$$F(u,\lambda,u_1,u_2,u_3) = \begin{bmatrix} G + ((u_2, D_uGu_1))u_2 \\ ((u_2, D_uGu_3 + D_\lambda G)) \\ D_uGu_1 + \frac{1}{2}[((u_1,u_1)) - 1]u_2 \\ D_uG^*u_2 + \frac{1}{2}[((u_2,u_2)) - 1]u_1 \\ D_uGu_3 + D_\lambda G + ((u_1,u_3))u_2 \end{bmatrix} \tag{2.1}$$

where $F: V\times R\times V\times V\times V \to V\times R\times V\times V\times V$ is well-defined.

Recalling the definition of $(u_0,\lambda_0,\phi,\psi,v_\lambda)$, for $x_0 = (u_0,\lambda_0,\phi,\psi,v_\lambda)$, we can get $F(x_0) = 0$, Simple calculation yields:

$$DF(x_0) = \begin{bmatrix} A & 0 & 0 & 0 \\ C & B & 0 & 0 \\ E & H & W & \end{bmatrix} \tag{2.2}$$

where

$$A := \begin{bmatrix} D_uG_0 + ((\psi, D_{uu}G_0\phi\cdot))\psi & D_\lambda G_0 + ((\psi, D_{u\lambda}G_0\phi))\psi \\ ((\psi, D_uDG_0(v_\lambda,1)\cdot)) & ((\psi, D_\lambda DG_0(v_\lambda,1))) \end{bmatrix} \tag{2.3}$$

$$B = D_u G_0 + ((\phi, \cdot))\psi; B^* = D_u G_0^* + ((\psi, \cdot))\phi; C = (D_{uu} G_0 \phi \cdot, D_{u\lambda} G_0 \phi) \tag{2.4}$$

$$E = \begin{bmatrix} D_{uu} G_0^* \psi \cdot & D_{u\lambda} G_0^* \psi \\ D_u D G_0(v_\lambda, 1) \cdot & D_\lambda D G_0(v_\lambda, 1) \end{bmatrix} \tag{2.5}$$

$$H = [0, ((\cdot, v_\lambda))\psi]^\top \tag{2.6}$$

$$W = \begin{bmatrix} B^* & 0 \\ 0 & B \end{bmatrix} \tag{2.7}$$

Lemma 2.1. *A, B, B^* are nonsingular.*

Proof. Firstly, we consider the homogeneous equation: $Bu = 0$, i.e.

$$D_u G_0 u + ((\phi, u))\psi = 0 \tag{2.8}$$

taking an inner product with ψ at the both sides of (2.8), one obtains, $((\phi, u)) = 0$, together with (2.8), we get $D_u G_0 u = 0$, i.e.$u = c\phi$. taking it back into $((\phi, u)) = 0$, which means $c = 0$. that implies the equation (2.8) has unique solution $u = 0$, thereafter B is nonsingular. similarly, we can prove the nonsingularity of B^*.

Now, we consider the following system:

$$A \begin{pmatrix} u \\ \lambda \end{pmatrix} = 0 \tag{2.9}$$

presisely

$$D_u G_0 u + ((\psi, D_{uu} G_0 \phi u))\psi + D_\lambda G_0 \lambda + \lambda((\psi, D_{u\lambda} G_0 \phi))\psi = 0 \tag{2.10}$$

$$((\psi, D_u D G_0(v_\lambda, 1)u)) + \lambda((\psi, D_\lambda D G_0(v_\lambda, 1))) = 0 \tag{2.11}$$

from (1.14) we derive

$$D_u G_0 u + D_\lambda G_0 \lambda = D_u G_0(u - \lambda v_\lambda)$$

thereafter, taking an inner product with ψ at the both sides of (2.10), we have

$$((\psi, D_{uu} G_0 \phi u + D_{u\lambda} G_0 \phi)) = 0 \tag{2.12}$$

taking it back into (2.10), we derive

$$D_u G_0(u - \lambda v_\lambda) = 0 \tag{2.13}$$

from (2.13), one obtains $u = \mu\phi + \lambda v_\lambda$ for $\lambda, \mu \in R$. taking it back into (2.12) and (2.11), we derive the system for $\mu, \lambda \in R$:

$$\begin{cases} \alpha\mu + \beta\lambda = 0 \\ \beta\mu + \gamma\lambda = 0 \end{cases} \tag{2.14}$$

due to $\beta^2 - \alpha\gamma > 0$, we get $\mu = \lambda = 0$, this yields that the equations (2.9) have trivial solution $\mu = \lambda = 0$ only. hence A is nonsingular.

If (u_0, λ_0) is transcritical nondegenerate simple bifurcation point, we can use the following lemma 2.2 to express the inverse of A explicitly.

Lemma 2.2. *assume that $\alpha \neq 0$, $M := D_u G_0 + \psi((\psi, D_{uu} G_0 \phi \cdot))$ is non singular, and*

$$M^{-1} = [I - \alpha^{-1}\phi(((\psi, D_{uu} G_0 \phi \cdot)) - ((\phi, \cdot)))]B^{-1} \tag{2.15}$$

Proof. Consider the homogeneous equation $Mu = 0$, i.e.

$$D_u G_0 u + \psi((\psi, D_{uu} G_0 \phi u)) = 0 \tag{2.16}$$

taking an inner product with ψ at its both sides, we derive:

$$((\psi, D_{uu}G_0\phi u)) = 0 \tag{2.17}$$

taking it back into (2.16), hence $D_uG_0u = 0$, i.e $u = c\phi$, together with (2.17) and $\alpha \neq 0$, we get $c = 0$. that means $u = 0$ is the unique solution of (2.16), for this reason M is nonsingular.

Now, we define function $V \to R$, $\theta := ((\psi, D_{uu}G_0\phi\cdot)) - ((\phi, \cdot))$. by virtue of (1.8), it is easy to verify that: $B\phi = \psi, B^{-1}\psi = \phi$, thereby

$$\theta B^{-1}\psi = \theta\phi = ((\psi, D_{uu}G_0\phi^2)) - ((\phi, \phi)) = \alpha - 1 \neq -1,$$

i.e.

$$\theta B^{-1}\psi + 1 = \alpha,$$

simple calculation yields

$$(I - \frac{B^{-1}\psi\theta}{1 + \theta B^{-1}\psi})B^{-1}(B + \psi\theta) = (B + \psi\theta)(I - \frac{B^{-1}\psi\theta}{1 + \theta B^{-1}\psi})B^{-1} = I$$

There we have

$$(B + \psi\theta)^{-1} = (I - \frac{B^{-1}\psi\theta}{1 + \theta B^{-1}\psi})B^{-1}$$

precisely, we get,

$$M^{-1} = [I - \alpha^{-1}\phi(((\psi, D_{uu}G_0\phi\cdot)) - ((\phi, \cdot)))]B^{-1}$$

In order to describe the inverse of A explicitly, we set

$$A^{-1} := \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{bmatrix}$$

It is follows that

$$\begin{cases} [D_uG_0 + ((\psi, D_{uu}G_0\phi\cdot))\psi]\tilde{a}_{11} + [D_\lambda G_0 + ((\psi, D_{u\lambda}G_0\phi))\psi]\tilde{a}_{21} = I \\ [D_uG_0 + ((\psi, D_{uu}G_0\phi\cdot))\psi]\tilde{a}_{12} + [D_\lambda G_0 + ((\psi, D_{u\lambda}G_0\phi))\psi]\tilde{a}_{22} = 0 \\ ((\psi, D_uDG_0(v_\lambda, 1)\cdot))\tilde{a}_{11} + ((\psi, D_\lambda DG_0(v_\lambda, 1)))\tilde{a}_{21} = 0 \\ ((\psi, D_uDG_0(v_\lambda, 1)\cdot))\tilde{a}_{12} + ((\psi, D_\lambda DG_0(v_\lambda, 1)))\tilde{a}_{22} = 1 \end{cases}$$

With complex calculation, we obtain:

$$\tilde{a}_{11} = [I - d^{-1}(\beta\phi - \alpha v_\lambda)((\psi, D_uDG_0(v_\lambda, 1)\cdot))]M^{-1} \tag{2.18}$$

$$\tilde{a}_{12} = -\alpha d^{-1}(v_\lambda - \beta M^{-1}\psi) \tag{2.19}$$

$$\tilde{a}_{21} = \alpha d^{-1}((\psi, D_uDG_0(v_\lambda, 1)\cdot))M^{-1} \tag{2.20}$$

$$\tilde{a}_{22} = -\alpha d^{-1} \tag{2.21}$$

Theorem 2.1. *$x_0 = (u_0, \lambda_0, \phi, \psi, v_\lambda)$ defined by $(H_1)(H_3)(1.8)(1.9)$ is the nonsingular solution of $F(x) = 0$.*

Proof. Obviously, $F(x_0) = 0$.

On the other hand, $DF(x_0)$ is a diagonal form. by virtue of Lemma 2.1, we can prove $DF(x_0)$ is nonsingular. that means x_0 is the nonsingular solution of $F(x) = 0$.

3. Newton-like Method for Transcritical Nondegenerate Simple Bifurcation Point

Although $x_0 = (u_0, \lambda_0, \phi, \psi, v_\lambda)$ is the nonsingular solution of $F(x) = 0$, scale of extended system (2.1) is bigger than (1.3), computation work is bigger too. due to the block structure of $D\,F_0$, we introduce a Newton-like method which allows reductions of computation complexity.

Let $x^0=(u^0,\lambda^0,u_1^0,u_2^0,u_3^0)$ be a starting vector near x_0, for $k=0,1,\cdots$, if x^{k-1} is known, x^k can be found from the followings:

$$\begin{cases} D(x^k)\cdot\Delta x^k=-F(x^k) \\ x^{k+1}=x^k+\Delta x^k \end{cases} \tag{3.1}$$

$$D(x^k)=\begin{bmatrix} A^k & 0 & 0 & 0 \\ C^k & B^k & 0 & 0 \\ E^k & H^k & W^k & \end{bmatrix}$$

where

$$A^k:=\begin{bmatrix} D_uG^k+((u_2^k,D_{uu}G^ku_1^k\cdot))u_2^k & D_\lambda G^k+((u_2^k,D_{u\lambda}G^ku_1^k))u_2^k \\ ((u_2^k,D_uD\,G^k(u_3^k,1)\cdot)) & ((u_2^k,D_\lambda DG^k(u_3^k,1))) \end{bmatrix} \tag{3.2}$$

$$B^k:=D_uG^k+((u_1^k,\cdot))u_2^k,\quad B^{*^k}:=D_uG^{*^k}+((u_2^k,\cdot))u_1^k \tag{3.3}$$

$$C^k:=(D_{uu}G^ku_1^k\cdot,D_{u\lambda}G^ku_1^k)=(C_1^k,C_2^k) \tag{3.4}$$

$$W^k:=\operatorname{diag}\{B^{*^k},B^k\}$$

$$E^k:=\begin{bmatrix} D_{uu}G^{*^k}u_2^k\cdot & D_{u\lambda}G^{*^k}u_2^k \\ D_uDG^k(u_3^k,1)\cdot & D_\lambda DG^k(u_3^k,1) \end{bmatrix}=\begin{bmatrix} E_{11}^k & E_{12}^k \\ E_{21}^k & E_{22}^k \end{bmatrix} \tag{3.5}$$

$$H^k:=[\frac{1}{2}(((u_2^k,u_2^k))-1),((\cdot,u_3^k))u_2^k]^\top=(H_1^k,H_2^k)^\top \tag{3.6}$$

Note, in discrete case $B^{*^k}=B^{k^\top}$

Denote: $F(x)=(f_1^k,f_2^k,f_3^k,f_4^k,f_5^k)^\top$

The system (3.1) can be written in block form:

$$\begin{aligned} &a)\quad A^k\cdot\begin{bmatrix}\Delta u^k\\ \Delta\lambda^k\end{bmatrix}=-\begin{bmatrix}f_1^k\\ f_2^k\end{bmatrix} \\ &b)\quad C^k\cdot\begin{bmatrix}\Delta u^k\\ \Delta\lambda^k\end{bmatrix}+B^k\cdot\Delta u_1^k=-f_3^k \\ &c)\quad E^k\cdot\begin{bmatrix}\Delta u^k\\ \Delta\lambda^k\end{bmatrix}+H^k\cdot\Delta u_1^k+W^k\cdot\begin{bmatrix}\Delta u_2^k\\ \Delta u_3^k\end{bmatrix}=-\begin{bmatrix}f_4^k\\ f_5^k\end{bmatrix} \end{aligned} \tag{3.7}$$

Algorithm is following:

1) Compute LU-decomposition for B^k
2) Compute
   $\alpha^k=((u_2^k,D_{uu}G^ku_1^ku_1^k))=2a(u_1^k,u_1^k,u_2^k)$
   $\beta^k=((u_2^k,D_uDG^k(u_3^k,1)u_1^k))=a_0(u_1^k,u_2^k)+a(u_1^k,u_3^k,u_2^k)+a(u_3^k,u_1^k,u_2^k)$
   $\gamma^k=((u_2^k,D^2G^k(u_3^k,1)^2))=2a(u_3^k,u_3^k,u_2^k)+2a_0(u_3^k,u_2^k)$
   $d^k=\beta^k\beta^k-\alpha^k\gamma^k$
   $\tilde a_{11}^k=[I-d^{-k}(u_1^k\beta^k-\alpha^ku_3^k)((u_2^k,D_uDG(u_3^k,1)\cdot))M^{-k}$
   $\tilde a_{12}^k=-\alpha^kd^{-k}(u_3^k-\beta^kM^{-k}u_2^k)$
   $\tilde a_{21}^k=\alpha^kd^{-k}((u_2^k,D_uDG(u_3^k,1)\cdot))M^{-k}$
   $\tilde a_{22}=-\alpha^kd^{-k}$
   $M^{-k}=[I-\alpha^{-k}u_1^k(((u_2^k,D_{uu}G^ku_1^k\cdot))-((u_1^k,\cdot)))]B^{-k}$
3) Compute $\Delta u^k,\Delta\lambda^k$:

$\Delta u^k = -\tilde{a}_{11}^k f_1^k - \tilde{a}_{12}^k f_2^k, \qquad \Delta\lambda^k = -\tilde{a}_{21}^k f_1^k - \tilde{a}_{22}^k f_2^k$

4) Compute Δu_1^k :

$\Delta u_1^k = -(B^k)^{-1} \cdot [f_3^k - C_1^k \Delta u^k - C_2^k \Delta\lambda^k]$

5) Compute $\Delta u_2^k, \Delta u_3^k$:

$\Delta u_2^k = -(B^{*^k})^{-1}(f_4^k - E_{11}^k \Delta u^k - E_{12}^k \Delta\lambda^k - H_1^k \Delta u_1^k)$

$\Delta u_3^k = -(B^k)^{-1}(f_5^k - E_{21}^k \Delta u^k - E_{22}^k \Delta\lambda^k - H_2^k \Delta u_1^k)$

Remark. The main work of our calculation is once LU-decomposition of B^k, five times back substitution, and algebraic calculation. that greatly reduces the computational complexity . in fact, computational complexity of splitting iterative method (3.1) is the same as computation complexity of (1.3), and we have the following conclusion.

Theorem 3.1. *The iteration algorithm (3.7) is quadratic converges.*

Proof: Define

$$\Phi(x) = x - D^{-1}(x)F(x) : V \times R \times V \times V \times V \to V \times R \times V \times V \times V,$$

then $\Phi(x_0) = 0, D\Phi(x_0) = 0$.

Taylor's formular yields.

$$\Phi(x) - x_0 = \Phi(x) - \Phi(x_0) = \int_0^1 (1-t)D^2\Phi(x + t(x - x_0))(x - x_0)^2 \mathrm{d}t$$

from the fixed point theorem, we obtain the local quadratic convergence of (3.7).

4. Numerical Experiment

The numerical test is carried out in axisymmetric spherical couette flow between two concentric different rotating spheres by using finite element method and described the splitting iteration method, namely, Newton-like method. Spherical Couette flow depend on three parameters, $\lambda = Re^{-1}, \varepsilon$ and η which are describing Reynolds number, the difference of angular velocities of two spheres and the gap of spheres. if we fix $\eta = 0.14$ and $\varepsilon = 0.9$, the $\lambda = Re^{-1}$ is taken bifurcation parameter. and we use the solution of the related Stokes equations as initial value, the splitting iteration method was applied to Compute transcritical nondegenerate simple bifurcation point. the numerical results are $Re = 642.05, Re = 739.08$. when $Re = 739.08$, we present the comparative results between Newton-like method (NLM) and old method (OM) in the following table. the first two rows indicate the error $||x_0 - x_0^k||$ and the rest two rows indicate the CPU time used by the two algorithms with respect to different iteration times k.

k	5	10	15	20
OM(error)	1.73-E3	1.69-E3	1.38-E3	1.27-E3
NLM(error)	1.81-E3	1.54-E3	1.37-E3	1.14-E3
OM(time)	127.8s	263.7s	541.2s	753.6s
NLM(time)	85.7s	91.3s	103.7s	187.5s

It is obvious that NLM can save a lot of CPU time. in the following Fig. 1-2, we give Streamlinear on the meridional plane

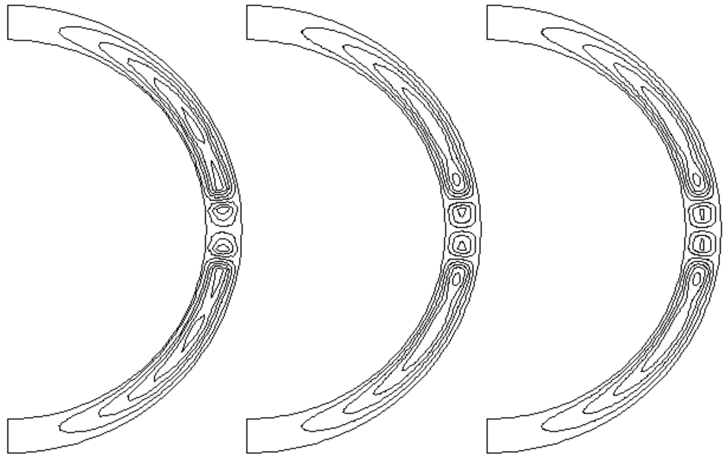

Fig.1 Streamlinear on the meridional plane, Re=642, 645, 650

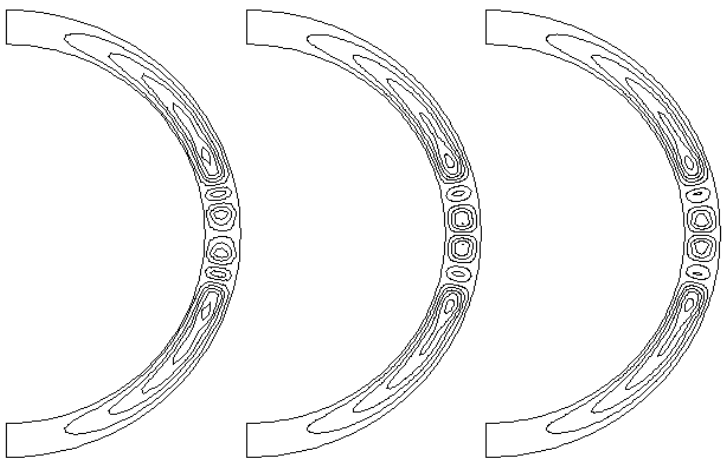

Fig.2 Streamlinear on the meridional plane, Re=739, 740, 745

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