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Multilinear Commutators of θ-Type Calderón-Zygmund Operators on Non-Homogeneous Metric Measure Spaces

Rulong Xie[1], Lisheng Shu[2] and Hailian Wang[1,*]

[1] *Department of Mathematics, Chaohu University, Hefei 238000, Anhui, China*
[2] *Department of Mathematics, Anhui Normal University, Wuhu 241000, Auhui, China*

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Abstract. In this paper, the boundedness in Lebesgue spaces of commutators and multilinear commutators generated by θ-type Calderón-Zygmund operators with $RBMO(\mu)$ functions on non-homogeneous metric measure spaces is obtained.

Key Words: Multilinear commutators, θ-type Calderón-Zygmund operators, non-homogeneous metric measure spaces, $RBMO(\mu)$.

AMS Subject Classifications: 42B20, 42B25

1 Introduction

It is well known that non-homogeneous metric measure spaces, which includes both the homogeneous spaces and non-doubling measure spaces, are introduced by Hytönen [8]. From then on, the properties for operators and function spaces in this background are obtained by many researchers. Hytönen et al. [11] and Bui and Duong [1] introduced independently the atomic Hardy space $H^1(\mu)$ and proved that the dual space of $H^1(\mu)$ is $RBMO(\mu)$. Bui and Duong [1] also proved that Calderón-Zygmund operator and commutators are bounded in $L^p(\mu)$ for $1<p<\infty$. Hytönen and Martikainen [9] established Tb theorem on non-homogeneous metric measure spaces. Boundedness of multilinear commutators of Calderón-Zygmund operators on Orlicz space is established by Fu et al. [4]. Recently, Morrey spaces and H^p spaces in this settings are also obtained by Cao and Zhou [2] and Fu et al. [3] respectively. We [6,19] obtained the boundedness for commutators of multilinear Calderón-Zygmund operators and multilinear fractional integral operators. For more results, one can refer to [5,7,10–15,22] and the references therein.

*Corresponding author. *Email addresses:* `rulongxie@163.com` (R. L. Xie), `shulsh@ahnu.edu.cn` (L. S. Shu), `hailianmath@163.com` (H. L. Wang)

In 1985, Yabuta [21] first introduced θ-type Calderón-Zygmund operator. Later, the properties of this operator are further studied by many researchers. We [20] obtained the boundedness of θ-type Calderón-Zygmund operators and commutators on non-doubling measure spaces. Ri and Zhang [16] researched the boundedness of θ-type Calderón-Zygmund operators on non-homogeneous metric measure spaces. Zheng et al. [23, 24] obtained some properties for bilinear θ-type Calderón-Zygmund operators and maximal bilinear θ-type Calderón-Zygmund operators.

In this paper, the boundedness for commutators and multilinear commutators generated by θ-type Calderón-Zygmund operators with $RBMO(\mu)$ functions on non-homoge -neous metric measure spaces is obtainded. This result includes corresponding results on both the homogeneous spaces and $(\mathbb{R}^n,\mu)$ with non-doubling measure spaces.

Throughout this paper, C always denotes a positive constant independent of the main parameters involved, but it may be different from line to line. And p' is the conjugate index of p, i.e., $1/p+1/p'=1$. Now, let us recall some definitions and notations.

Definition 1.1 (see [8])**.** A metric space (X,d) is geometrically doubling if there exists some $N_0\in\mathbf{N}$ such that, for every ball $B(x,r)\subset X$, there exists a finite ball covering $\{B(x_i,r/2)\}_i$ of $B(x,r)$ such that the cardinality of this covering is at most N_0.

Definition 1.2 (see [8])**.** A metric measure space (X,d,μ) is upper doubling if μ is a Borel measure on X and there exists a function $\lambda: X\times(0,+\infty)\to(0,+\infty)$ and a constant $C_\lambda>0$ such that for every $x\in X, r\longmapsto(x,r)$ is non-decreasing, and for any $x\in X, r>0$,

$$\mu(B(x,r))\le\lambda(x,r)\le C_\lambda\lambda(x,r/2). \tag{1.1}$$

Remark 1.1. (i) A homogeneous space is an upper doubling space, if we take $\lambda(x,r)=\mu(B(x,r))$. Also, a non-doubling measure space, satisfying the following polynomial growth condition:

$$\mu(B(x,r))\le Cr^n \tag{1.2}$$

for all $x\in\mathbb{R}^d$ and $r>0$, is also an upper doubling measure space if we take $\lambda(x,r)=Cr^n$.

(ii) It was shown in [11] that there exists another function $\tilde{\lambda}$ such that for any $x,y\in X$, $d(x,y)\le r$,

$$\tilde{\lambda}(x,r)\le\tilde{C}\tilde{\lambda}(y,r). \tag{1.3}$$

Thus, one assumes that λ always satisfies (1.3) in this paper.

(iii) Tan and Li [17] pointed that the upper doubling condition is equivalent to the weak growth condition.

Let $1\le\alpha,\beta\le+\infty$, if $\mu(\alpha B)\le\beta\mu(B)$, then a ball $B\subset X$ is called to be (α,β)-doubling. By Lemma 2.3 of [1], we know that there exist plenties of (α,β)-doubling balls with small radii and with large radii. Unless α and β are specified, otherwise in this paper one means (α,β)-doubling ball is $(6,\beta_0)$-doubling with $\beta_0>\max\{C_\lambda^{3\log_2 6},6^n\}$, where $n=\log_2 N_0$ is the geometric dimension of the space.