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Computational Aspect For Function-Valued Padé-Type Approximation

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Abstract

The computational problems of two special determinants are investigated. Those determinants appear in the construction of the function-valued Padé-type approximation for computing Fredholm integral equation of the second kind. The main tool to be used in this paper is the well-known Schur complement theorem.

Keywords: Function-valued Padé-type approximation; Fredholm integral equation of the second kind; determinant; Schur complement.
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1. Introduction

Consider a Fredholm integral equation of the second kind

$$x(s) = y(s) + \lambda \int_a^b K(s,t)x(t)dt, \quad a \le s, \quad t \le b, \tag{1.1}$$

where $K(s,t)$ and $y(s)$ are both continuous functions in the square area $[a,b] \times [a,b]$ and the interval $[a,b]$, respectively. The solution of equation (1.1) can be expressed as a power series with function-valued coefficient

$$x(s) = f(s,\lambda) = y_0(s) + y_1(s)\lambda + y_2(s)\lambda^2 + \cdots + y_n(s)\lambda^n + \cdots, \tag{1.2}$$

where

$$y_i(s) = \int_a^b K^i(s,t)y(t)dt, \quad i \ge 1, \quad y_0(s) = y(s), \tag{1.3}$$

and $K^i(s,t)$ in (1.3) is called the i-th iterative kernel.

Suppose that $x(s) = f(s,\lambda)$ as a function about λ is analytic when $\lambda = 0$, so the series (1.2) is convergent when $|\lambda|$ is sufficiently small. Meanwhile, $y_i(s)$ is a continuous function in the interval $[a,b]$. For $y_i(s),\ y_j(s) \in L^2[a,b]$, define the inner product by

$$(y_i, y_j) = (y_i(s), y_j(s)) = \int_a^b y_i(s)y_j(s)ds, \tag{1.4}$$

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and define the corresponding norm by

$$\| y_i(s) \| = \sqrt{(y_i(s), y_i(s))} = \left\{ \int_a^b y_i^2(s) ds \right\}^{\frac{1}{2}}.$$

A function-valued Padé-type approximation of type (m/n) for the power series (1.2) is denoted by $(m/n)_f(s,\lambda)$.

Theorem 1.1. ([1,2]) *If* $\mathbf{det}(A_n) \neq 0$*, then* $(m/n)_f(s,\lambda)$ *exists, and it holds that*

$$(m/n)_f(s,\lambda) = p_{m,n}(s,\lambda)/q_{m,n}(\lambda),$$

with

$$q_{m,n}(\lambda) = \mathbf{det}\begin{bmatrix} A_n & l^{(n)} \\ \lambda^{(n)^T} & 1 \end{bmatrix}, \tag{1.5}$$

$$p_{m,n}(s,\lambda) = \mathbf{det}\begin{bmatrix} A_n & l^{(n)} \\ \tilde{\lambda}^{(n)^T} & \eta \end{bmatrix}, \tag{1.6}$$

where

$$A_n = \begin{bmatrix} (y_{m-n+1}, y_{m-n+1}) & (y_{m-n+1}, y_{m-n+2}) & \cdots & (y_{m-n+1}, y_m) \\ (y_{m-n+1}, y_{m-n+2}) & (y_{m-n+1}, y_{m-n+3}) & \cdots & (y_{m-n+1}, y_{m+1}) \\ \cdots & \cdots & \cdots & \\ (y_{m-n+1}, y_m) & (y_{m-n+1}, y_{m+1}) & \cdots & (y_{m-n+1}, y_{m+n-1}) \end{bmatrix}, \tag{1.7}$$

$$\begin{aligned} \lambda^{(n)} &= (\lambda^n, \lambda^{n-1}, \cdots, \lambda)^T, \\ \tilde{\lambda}^{(n)} &= \left(\sum_{i=n}^m y_{i-n}\lambda^i, \sum_{i=n-1}^m y_{i-n+1}\lambda^i, \cdots, \sum_{i=1}^m y_{i-1}\lambda^i \right)^T, \\ l^{(n)} &= \big((y_{m-n+1}, y_{m+1}), (y_{m-n+1}, y_{m+2}), \cdots, (y_{m-n+1}, y_{m+n}) \big)^T, \\ \eta &= \sum_{i=0}^m y_i \lambda^i. \end{aligned}$$

From Theorem 1.1, we observe that the central point to construct a $(m/n)_f(s,\lambda)$ is how to compute two determinants (1.5) and (1.6). Therefore, we need the following well-known result.

Lemma 1.1 (Schur complement)**.** *Let* A *be an* $n \times n$ *real matrix and partitioned into* 2×2 *block matrix* $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$. *If* $A_{11} \in C^{k\times k}$ *is nonsingular, then*

$$\mathbf{det}(A) = \mathbf{det}(A_{11})\mathbf{det}(A_{22} - A_{21}A_{11}^{-1}A_{12}).$$