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SOME PROPERTIES OF CENTROSYMMETRIC MATRICES AND ITS APPLICATIONS*

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Abstract *In this paper, some properties of centrosymmetric matrices, which often appear in the construction of orthonormal wavelet basis in wavelet analysis, are investigated. As an application, an algorithm which is tightly related to a so-called Lawton matrix is presented. In this algorithm, about only half of memory units are required and quarter of computational cost is needed by exploiting the property of the Lawton matrix and using a compression technique, it is compared to one for the original Lawton matrix.*

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1 Introduction

Wavelet analysis is a rapidly developing area in the mathematical science which is emerging as a brisk and important field of investigation. Moreover, it has already created a tight link between mathematic scientists and electrical engineers, and has attracted a great deal of attention from scientists and engineers in other disciplines.

Historically, the study of wavelet analysis was based on the standard approach of functional analysis. However, the basic aspects of wavelet analysis can be derived using fairly elementary means of matrix algebra, and matrix methods play an important role in the study of wavelets, see, for example, the references [2, 5-8, 10].

In this paper, we first investigate some properties of a kind of special matrices, which are called centrosymmetric matrices, they appear frequently in the construction of orthonormal wavelet basis. Then we develop an algorithm tightly related to a so-called Lawton matrix−a

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special centrosymmetric matrix, which is used to justify the orthogonality of the compactly supported wavelet bases. By exploiting the special property and using a compression technique, in particular, our algorithm needs only about half of memory units and quarter of computational costs. It is compared to one for the original Lawton matrix.

2 Some Properties of Centrosymmtry-like Matrices

In this section we'll introduce some basic concept and present some useful results used in the sequel. We begin with some basic notation which refers to the definition of a symmetric matrix, skew symmetric matrix, Hermitian matrix, skew Hermitian matrix, respectively.

Let a matrix $A = (a_{i,j})_{p\times q}$ be a $p \times q$ complex matrix, then the matrix A is called:

1. a centrosymmetric matrix, if the elements of A satisfy the relation

$$a_{i,j} = a_{p-i+1,q-j+1};$$

2. a skew centrosymmetric matrix, if the elements of A satisfy the relation

$$a_{i,j} = -a_{p-i+1,q-j+1};$$

3. a centrohermitian matrix, if the elements of A satisfy the relation

$$a_{i,j} = \overline{a_{p-i+1,q-j+1}};$$

4. a skew centrohermitian matrix, if the elements of A satisfy the relation

$$a_{i,j} = -\overline{a_{p-i+1,q-j+1}},$$

respectively, for all $1 \le i \le p$ and $1 \le j \le q$.

For the centrosymmetric matrices, one can easily check the following properties.

Theorem 1

1. (a) Let the matrix $C \in \mathbf{C}^{n\times n}$ be centrosymmetric, skew centrosymmetric, centrohermitian, skew centrohermitian, respectively. If C is nonsingular, then C^{-1} is centrosymmetric, skew centrosymmetric,centrohermitian, skew centrohermitian, respectively; (b) If the matrix $H \in \mathbf{C}^{p\times q}$ be centrosymmetric, skew centrosymmetric, centrohermitian, skew centrohermitian, respectively, then H^T is $q \times p$ centrosymmetric, skew centrosymmetric, centrohermitian, skew centrohermitian, respectively; (c) If both the matrices E and $F \in \mathbf{C}^{n\times l}$ be centrosymmetric, skew centrosymmetric, skew centrohermitian respectively, then $E \pm F$ are centrosymmetric, skew centrosymmetric,centrohermitian, skew centrohermitian respectively.

2. If both the matrices $A \in \mathbf{C}^{n\times k}$ and $B \in \mathbf{C}^{k\times l}$ be centrosymmetric, skew centrosymmetric, respectively, then $AB \in \mathbf{C}^{n\times l}$ is centrosymmetric.