English Series | NUMERICAL MATHEMATICS A Journal of Chinese Universities | Vol.14, No.3 Aug. 2005

AN ITERATIVE BEM FOR THE INVERSE PROBLEM OF DETECTING CORROSION IN A PIPE*

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Abstract *In this paper, we consider an inverse problem of determining the corrosion occurring in an inaccessible interior part of a pipe from the measurements on the outer boundary. The problem is modelled by Laplace's equation with an unknown term* γ *in the boundary condition on the inner boundary. Based on the Maz'ya iterative algorithm, a regularized BEM method is proposed for obtaining approximate solutions for this inverse problem. The numerical results show that our method can be easily realized and is quite effective.*

Key words *Corrosion, iterative method, Cuachy problem, Laplace equation.*

AMS(2000)subject classifications 35B30, 35R30, 65M32

1 Introduction

Detecting the corrosion inside a pipe is one of the most important topics in the engineering, especially for the safety administration of the nuclear power station. There are several ways to do this. In this paper, we will discuss the mathematical theory and numerical algorithm for a method of detecting the corrosion by electrical fields. More exactly, we consider an inverse problem of determining the corrosion occurring in an inaccessible interior part of a pipe from the measurements on the outer boundary. Our goal is to determine information about the corrosion that possibly occurs on an interior surface of the pipe, which is an 'inaccessible' part, and we collect electrostatic data on the part of the exterior surface of the pipe, which is an 'accessible' part.

In the case that the thickness of the pipe is sufficient small compared with the radius of the

* The research is partly supported by NSF of China (No. 10271032) and Shuguang Project from the Education Commission of Shanghai, China.
Received: Sep. 1, 2004.

pipe and the Cauchy data are given on the whole outer boundary, this inverse problem can be treated by Thin Plate Approximation method (TPA). The algorithm and numerical analysis can be found in [7]. But this algorithm works only under the assumption that the thickness is small enough compared with the radius of the pipe. The case, in which Cauchy data are given on the part of the outer boundary and the smallness assumption is abandoned, has not been studied and it is obvious that is of great important for the practice problems.

The main difficulty for this inverse problem is ill-posedness of the inverse problem. The measured data are given only on part of the outer boundary and we want to determine an unknown function in the inner boundary. Because of the ill-posedness, the errors in measured data will be enlarged in the numerical treatment if we do not treat it suitably. In this paper, based on the Maz'ya iterative method, we propose a new BEM algorithm for this inverse problem. It can easily realized. The numerical results show the efficiency of this method.

This paper is organized as the follows: In section 1 the formulation of the inverse problem will be given; In section 2 we will introduce the iterative boundary element method, and some numerical examples based on it will be present in section 3. Finally what in section 4 is our conclusion.

2 Formulation of the inverse problem

Suppose that a domain $\Omega = \{x \,|\, r_1 < |x| < r_2\} \subset \mathbb{R}^2$ (see figure 1) and the boundaries $\Gamma_1 = \{x|\,|x| = r_1\}$, $\Gamma_2 = \{x|\,|x| = r_2\}$.

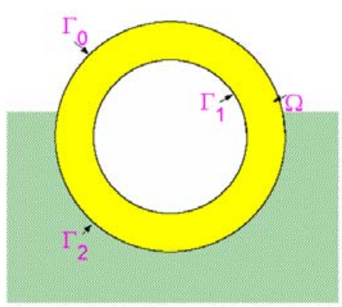

figure 1

Assume that Ω is a metallic body with constant conductivity. In the domain Ω, we consider the electrostatic field. The electric potential u satisfies the Laplace's equation in Ω, i.e.,

$$\Delta u = 0, \qquad \text{in} \quad \Omega. \tag{2.1}$$

Let Γ_0 be the open set of outer boundary Γ_2 of Ω which is an 'accessible' part. On Γ_0, the