Analysis in Theory and Applications
Anal. Theory Appl., Vol. **33**, No. 3 (2017), pp. 206-218

DOI: 10.4208/ata.2017.v33.n3.2

Dynamics of a Mutualistic Model with Diffusion

Mei Li*

School of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing 210023, China

Received 12 January 2016; Accepted (in revised version) 15 December 2016

Abstract. This article is concerned with a system of semilinear parabolic equations with no-flux boundary condition in a mutualistic ecological model. Stability result of the equilibrium about relevant ODE problem is proved by discussing its Jacobian matrix, we give two priori estimates and prove that the model is permanent when $\varepsilon_1+\varepsilon_2\neq 0$. Moreover sufficient conditions for the global asymptotical stability of the unique positive equilibrium of the model are obtained. Nonexistence of nonconstant positive steady states of the model is also given. When $\varepsilon_1+\varepsilon_2=0$, grow up property is derived if the geometric mean of the interaction coefficients is greater than 1 ($\alpha_1\alpha_2>1$), while if the geometric mean of the interaction coefficients is less than 1 ($\alpha_1\alpha_2<1$), there exists a global solution. Finally, numerical simulations are given.

Key Words: Reaction-diffusion systems, mutualistic model, equilibrium, permanence, grow-up.

AMS Subject Classifications: 35R35, 35K60

1 Introduction

Mutualism is an important biological interaction in nature. It occurs when one species provides some benefit in exchange for some benefit, for example, pollinators and flowering plants, the pollinators obtain floral nectar (and in some cases pollen) as a food resource while the plant obtains non-trophic reproductive benefits through pollen dispersal and seed production. Another instance is ants and aphids, in which the ants obtain honeydew food resources excreted by aphids while the aphids obtain increased survival by the non-trophic service of ant defense against natural enemies of the aphids. Lots of authors have discussed these models [1,2,4,6–10,14,17]. One of the simplest models is the classical Lotka-Volterra two-species cooperating model as follow:

$$\begin{cases} \dot{x}(t)=x(t)\big(a_1-b_1x(t)+c_1y(t)\big), \\ \dot{y}(t)=y(t)\big(a_2+b_2x(t)-c_2y(t)\big). \end{cases} \tag{1.1}$$

*Corresponding author. *Email address:* `limei@njue.edu.cn` (M. Li)

Among various types mutualistic model, we should specially mention the following model which was proposed by May (see [16]) in 1976:

$$\begin{cases} \dot{u}(t)=r_1u\Big(1-\dfrac{u}{K_1+\alpha_1 v}-\varepsilon_1 u\Big), \\ \dot{v}(t)=r_2v\Big(1-\dfrac{v}{K_2+\alpha_2 u}-\varepsilon_2 v\Big), \end{cases} \tag{1.2}$$

where r_i, K_i, α_i, $(i=1,2)$ are positive constants and ε_i, $(i=1,2)$ are nonnegative constants. Inspired by the former work, we will study the following mutualistic model in this paper.

$$\begin{cases} \dfrac{\partial u(t,x)}{\partial t}=a\Delta u+r_1u\Big(1-\dfrac{u}{K_1+\alpha_1 v}-\varepsilon_1 u\Big), & t>0, \quad x\in\Omega, \\ \dfrac{\partial v(t,x)}{\partial t}=b\Delta v+r_2v\Big(1-\dfrac{v}{K_2+\alpha_2 u}-\varepsilon_2 v\Big), & t>0, \quad x\in\Omega, \\ \dfrac{\partial u}{\partial \nu}=\dfrac{\partial v}{\partial \nu}=0, & t>0, \quad x\in\partial\Omega, \\ u(0,x)=u_0(x), \quad v(0,x)=v_0(x), & x\in\Omega, \end{cases} \tag{1.3}$$

where ν is the outward unit normal vector of the boundary $\partial\Omega$, positive constants a and b are the diffusion coefficient and the initial functions u_0 and v_0 are nonnegative and continuous, we can refer to [13] and the references therein.

The remainder of this paper is organized as follows. In the next section, stability result of the equilibrium about relevant ODE problem (1.3) is proved by discussing its Jacobian matrix. In Section 3, we give two priori estimates to prove the system of (1.3) is permanent when $\varepsilon_1+\varepsilon_2\neq 0$. Section 4 is devoted to sufficient conditions for the global asymptotical stability of the unique positive equilibrium of (1.3). In Section 6, when $\varepsilon_1+\varepsilon_2=0$, we derive its grow-up property if $\alpha_1\alpha_2>1$ and global property if $\alpha_1\alpha_2<1$. Finally, numerical simulations are given and a brief discussion is also given.

2 Stability analysis of relevant ODE system

In this section, we discuss the relevant ODE system of (1.3) and stability analysis of its equilibria is given.

Theorem 2.1. (i) $E_1=(0,0)$ is an unstable node;
(ii) $E_2=(0,\frac{K_2}{1+\varepsilon_2K_2})$ and $E_3=(\frac{K_1}{1+\varepsilon_1K_1},0)$ are saddle points;
(iii) The nontrivial equilibrium point $E_4=(u^*,v^*)$ of (1.2) is locally asymptotically stable.

Proof. We only prove (iii) since the proof of (i) and (ii) are similar.

The Jacobian matrix of system (1.2) at $E_4=(u^*,v^*)$ is

$$J_{E_4}=\begin{pmatrix} -r_1 & \alpha_1 r_1(1-\varepsilon_1u^*)^2 \\ \alpha_2 r_2(1-\varepsilon_2v^*)^2 & -r_2 \end{pmatrix}. \tag{2.1}$$