Numer. Math. Theor. Meth. Appl. Vol. **11**, No. 1, pp. 169-186
doi: 10.4208/nmtma.OA-2017-0019 February 2018

# Rational Quasi-Interpolation Approximation of Scattered Data in $\mathbb{R}^3$

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Received 17 February 2017; Accepted (in revised version) 14 March 2017

**Abstract.** This paper is concerned with a piecewise smooth rational quasi-interpolation with algebraic accuracy of degree $(n+1)$ to approximate the scattered data in $\mathbb{R}^3$. We firstly use the modified Taylor expansion to expand the mean value coordinates interpolation with algebraic accuracy of degree one to one with algebraic accuracy of degree $(n+1)$. Then, based on the triangulation of the scattered nodes in $\mathbb{R}^2$, on each triangle a rational quasi-interpolation function is constructed. The constructed rational quasi-interpolation is a linear combination of three different expanded mean value coordinates interpolations and it has algebraic accuracy of degree $(n+1)$. By comparing accuracy, stability, and efficiency with the $C^1$-Tri-interpolation method of Goodman[16] and the MQ Shepard method, it is observed that our method has some computational advantages.

**AMS subject classifications**: 41A05, 41A25

**Key words**: Scattered data, mean value coordinates interpolation, modified Taylor expansion, rational quasi-interpolation, algebraic accuracy.

## 1. Introduction

The problem of scattered data approximation appears in many fields of science and engineering. For example, geology, geography, reverse engineering, numerical simulation, computer graphics and geometric modeling, etc.. The most commonly used approximation method is the radial basis function interpolation [1-3], which is a kind of global interpolation method and need to solve linear system of equations to determine the coefficients of interpolation basis functions. The system is usually ill-conditioned when scattered data on a large scale, so they can't be solved effectively and stably. One of the ways to solve this problem is to find a better basis function, for example, the basis function in [4]. One way to get around this problem is the quasi-interpolation method. The quasi-interpolation

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method gives an explicit expression of the approximation function using the given data. Thus it avoids solving large-scale systems of linear algebraic equations in the radial basis function interpolation.

For a set of function values $\{f(\mathbf{v}_j)\}_{1\le j\le N}$ taken on a set of nodes $\Xi=\{\mathbf{v}_j\}_{1\le j\le N}\subset\mathbb{R}^d$, the form of quasi-interpolation function $\Phi(f;\mathbf{v})$ corresponding to $f(\mathbf{v})$ is

$$\Phi(f;\mathbf{v})=\sum_{j=1}^{N}f(\mathbf{v}_j)\varphi_j(\mathbf{v}),$$

where $\{\varphi_j\}_{1\le j\le N}$ is a set of quasi-interpolation basis functions. The set of nodes $\{\mathbf{v}_j\}_{1\le j\le N}$ usually has two kinds: the uniform grid node set and the scattered node set. The standard quasi-interpolant based on the uniform grid node set in $\mathbb{Z}^d$ is

$$\sum_{j\in\mathbb{Z}^d}f(jh)\varphi_{j,h}(\mathbf{v}), \tag{1.1}$$

in which Schoenberg model [5]

$$\sum f(jh)\Phi(\frac{\mathbf{v}}{h}-j)\sim f(\mathbf{v}),\quad \mathbf{v}\in\mathbb{R}^d \tag{1.2}$$

has attracted the most attention. Quasi-interpolant (1.2) can be studied via the theory of principal shift-invariant spaces, which has been developed in several articles by de Boor et al. [6,7]. Strang and Fix [8] also give a necessary and sufficient condition for the convergence of such a standard form of quasi-interpolant. The quasi-interpolants based on the uniform grid node set, have been applying in the numerical integration, the numerical solution of integral equation and the differential equation [9,10]. The quasi-interpolant (1.1) is based on the values of $f(\mathbf{v})$ in the uniform grid node set, which limits its range of application. For example, the above mentioned large-scale scattered data approximation, the numerical solution of integral equation and differential equation which are based on the non-uniform grid subdivision, and other solving problems. These problems can be solved, relying on the quasi-interpolants based on the scattered node set. The construction of the quasi-interpolants based on the high dimensional scattered node set, is firstly studied by Dyn and Ron [11]. They proposed the general idea about extending the quasi-interpolant based on the uniform grid node set to the scattered node set. Buhmann et al. [12] extended the scheme based on the uniform node set in [13] to the quasi-uniform distribution of the infinite scattered node set. By constructing the suitable "bell shape" basis function and using the convolution equation, Yoon [14] gives an integral form of the quasi-interpolant which is based on the scattered node set. The constructed quasi-interpolants based on the scattered node set in these papers not only need the function information at the scattered node but also need the function information at uniform node or all the information of the approximated function. This still limits the application of these methods. Wu and Liu [15] use the generalized Strang-Fix condition which is related to non-stationary quasi-interpolation, to extend their constructed quasi-interpolant based