Numer. Math. Theor. Meth. Appl.
doi: 10.4208/nmtma.OA-2017-0046

Vol. **11**, No. 1, pp. 49-73
February 2018

A Boosting Procedure for Variational-Based Image Restoration

Samad Wali[1], Zhifang Liu[1], Chunlin Wu[1] and Huibin Chang[2,*]

[1] *School of Mathematical Sciences, Nankai University, Tianjin, China*
[2] *School of Mathematical Sciences, Tianjin Normal University, Tianjin, China*

Received 12 July 2017; Accepted (in revised version) 13 July 2017

Abstract. Variational methods are an important class of methods for general image restoration. Boosting technique has been shown capable of improving many image denoising algorithms. This paper discusses a boosting technique for general variational image restoration methods. It broadens the applications of boosting techniques to a wide range of image restoration problems, including not only denoising but also deblurring and inpainting. In particular, we combine the recent SOS technique with dynamic parameter to variational methods. The dynamic regularization parameter is motivated by Meyer's analysis on the ROF model. In each iteration of the boosting scheme, the variational model is solved by augmented Lagrangian method. The convergence analysis of the boosting process is shown in a special case of total variation image denoising with a "disk" input data. We have implemented our boosting technique for several image restoration problems such as denoising, inpainting and deblurring. The numerical results demonstrate promising improvement over standard variational restoration models such as total variation based models and higher order variational model as total generalized variation.

AMS subject classifications: 68U10, 90C25, 49M37

Key words: Variational method, image restoration, total variation, boosting, augmented Lagrangian method.

1. Introduction

Image restoration is the operation of estimating the clean or original image from an input corrupted image. These operations, such as denoising, inpainting and deblurring, are the most fundamental tasks in image processing. Suppose we have an observed image f, which is degraded from the true image u. In many cases the degradation can be expressed as follows:

$$f = \mathscr{A}u + n, \tag{1.1}$$

*Corresponding author. *Email addresses:* `samad.walikhan@gmail.com` (S. Wali), `liuzhifang0628@163.com` (Z. F. Liu), `wucl@nankai.edu.cn` (C. L. Wu), `changhuibin@gmail.com` (H. B. Chang)

where $\mathscr{A}$ is a convolution operator and n is some random noise such as Gaussian noise or impulsive noise. In order to estimate u, it is necessary to solve the above inverse and ill-posed problem.

To figure out an approximation $\hat{u}$ of the original image u, a large number of variational models and algorithms based energy regularization have been developed. One of the most successful regularization is the total variation regularization [38], which can preserve image edges quite well. Total variation has extensive applications and benefit effective optimization algorithms [2, 10, 11, 26, 39, 48, 49, 51, 52]. It also has been extended to higher order and vectorial models [12,28,30,31,42,53,54] for gray scale and color image restoration. These models and algorithms rely on powerful image sparse representations [6, 19].

In spite of the performance and effectiveness of the above mentioned restoration algorithms, the result could be improved by applying a boosting technique. Boosting usually means a procedure calling an existing image processing algorithm iteratively, where the output of the current step is used as a part of input of the next step. This technique, to the best of our knowledge, has been only used in image denoising problem ($\mathscr{A} = I$ in Eq. (1.1)). We now review several existing boosting techniques. The "twicing method" [47] is very early and effective method which was proposed by Tukey. This method can be written as;

$$\hat{u}_{k+1} = B(f) + B(f - \hat{u}_k), \tag{1.2}$$

where $B(\cdot)$ represents the restoration algorithm and $\hat{u}_k$ is the kth iteration of denoised image. This concept was used in [29] to improve filters. Another interesting earlier work was given in [43], where the authors have proposed an iterative procedure based on the ROF model [38]. The procedure generates a sequence u_k which converges to the input image f. The procedure is stated as: $f = u^\lambda + v^\lambda$,

$$[u^\lambda, v^\lambda] = \arg\min_{u+v=f} J(f, \lambda; BV, L^2), \tag{1.3}$$

where λ is a weighting parameter serving as a scaling level to separate the two terms. This model was proposed for image decomposition based on hierarchical image representation of f. In [35] Osher et al proposed an iterative regularization method in which the residual was added back to the observed signal,

$$\hat{u}_{k+1} = B\Big(f + \sum_{i=1}^{k}(f - \hat{u}_k)\Big). \tag{1.4}$$

In [17] the authors have proposed Unsharp Residual Iteration (URI) method which is given by,

$$\hat{u}_{k+1} = \hat{u}_1 + (\hat{u}_k - B(\hat{u}_k)). \tag{1.5}$$

This method was applied for texture, grant manipulation and transfer. Another similar approach was used in [16], which can be expressed as:

$$\hat{u}_{k+1} = \hat{u}_k + B(f - \hat{u}_k). \tag{1.6}$$