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A Compact Third-Order Gas-Kinetic Scheme for Compressible Euler and Navier-Stokes Equations

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Abstract. In this paper, a compact third-order gas-kinetic scheme is proposed for the compressible Euler and Navier-Stokes equations. The main reason for the feasibility to develop such a high-order scheme with compact stencil, which involves only neighboring cells, is due to the use of a high-order gas evolution model. Besides the evaluation of the time-dependent flux function across a cell interface, the high-order gas evolution model also provides an accurate time-dependent solution of the flow variables at a cell interface. Therefore, the current scheme not only updates the cell averaged conservative flow variables inside each control volume, but also tracks the flow variables at the cell interface at the next time level. As a result, with both cell averaged and cell interface values, the high-order reconstruction in the current scheme can be done compactly. Different from using a weak formulation for high-order accuracy in the Discontinuous Galerkin method, the current scheme is based on the strong solution, where the flow evolution starting from a piecewise discontinuous high-order initial data is precisely followed. The cell interface time-dependent flow variables can be used for the initial data reconstruction at the beginning of next time step. Even with compact stencil, the current scheme has third-order accuracy in the smooth flow regions, and has favorable shock capturing property in the discontinuous regions. It can be faithfully used from the incompressible limit to the hypersonic flow computations, and many test cases are used to validate the current scheme. In comparison with many other high-order schemes, the current method avoids the use of Gaussian points for the flux evaluation along the cell interface and the multi-stage Runge-Kutta time stepping technique. Due to its multidimensional property of including both derivatives of flow variables in the normal and tangential directions of a cell interface, the viscous flow solution, especially those with vortex structure, can be accurately captured. With the same stencil of a second order scheme, numerical tests demonstrate that the current scheme is as robust as well-developed second-order shock capturing schemes, but provides more accurate numerical solutions than the second order counterparts.

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1 Introduction

Most computational fluid dynamics methods used in practical applications are second-order methods. They are generally robust and reliable. For the same computing cost, the high-order methods (order ≥ 3) can provide more accurate solutions, but they are less robust and more complicated. There has been a surge of research activities on the development of high-order methods for solving the Euler and Navier-Stokes equations. At the current stage, many high-order numerical methods have been developed, including discontinuous Galerkin (DG), spectral volume (SV), spectral difference (SD), correction procedure using reconstruction (CPR), essential non-oscillatory (ENO), and weighted essential non-oscillatory (WENO), etc. The DG scheme was first proposed in [29] to solve the neutron transport equation. A major development of the DG method was carried out by Cockburn et al. [4,5] to solve the hyperbolic conservation laws. In the DG method, high-order accuracy is achieved by means of high-order polynomial approximation within each element rather than by means of wide stencils, and Runge-Kutta method is used for the time discretization. Because only flow interaction from neighboring elements is included, it becomes compact and efficient in the application on complex geometry. Recently, a correction procedure via reconstruction framework (CPR) was developed by Wang et al. [12, 36]. This method was originally developed to solve one-dimensional conservation laws, under the name of flux reconstruction [14, 15]. Under lifting collocation penalty, the CPR framework was extended to two-dimensional triangular and mixed grids. The CPR formulation is based on a nodal differential form, with an element-wise continuous polynomial solution space. By choosing certain correction functions, the CPR framework can unify several well known methods, such as the DG, SV [23] and SD [35] methods and lead to simplified versions of these methods, at least for linear equations. The CPR method is compact because only immediate face neighbors play a role in updating the solutions in the current cell. Therefore, the complexity of implementation can be reduced, especially for the simulation with unstructured mesh. The main problem for the above DG-type schemes are the robustness of the method, especially in the cases with discontinuities. It is certainly true that the use of limiters can save the DG methods in the flow computations with discontinuities. But, the DG method is extremely sensitive to the limiters, because it is hard to distinguish the continuous or discontinuous solution in a computation, especially with the changing of cell size. Sometimes, the DG method can mysteriously get failure in a computation without clear reasons. Therefore, to pick up the trouble cells beforehand becomes a practice in the DG method. After so many years' research on the DG method, it gives perfect results for the continuous flow simulations, but seems have physical problem in its weak formulation in the discontinuous case.

The ENO scheme was proposed by Harten et al. [9, 31] and successfully applied to solve hyperbolic conservation laws and other convection dominated problems. Following the ENO scheme, WENO scheme [10,17,22] was further developed. ENO scheme uses the smoothest stencil among several candidates to approximate the numerical fluxes at cell interface for high-order accuracy. At the same time, it avoids spurious oscillations