J. Math. Study
doi: 10.4208/jms.v51n1.18.04

Vol. **51**, No. 1, pp. 57-75
March 2018

A Domain Decomposition Chebyshev Spectral Collocation Method for Volterra Integral Equations

Hua Wu*, Yunzhen Zhu, Hailu Wang and Lingfang Xu

Department of Mathematics, Shanghai University, Shanghai 200444, P.R. China.

Received August 28, 2017; Accepted January 16, 2018

Abstract. We develop a domain decomposition Chebyshev spectral collocation method for the second-kind linear and nonlinear Volterra integral equations with smooth kernel functions. The method is easy to implement and possesses high accuracy. In the convergence analysis, we derive the spectral convergence order under the L^∞-norm without the Chebyshev weight function, and we also show numerical examples which coincide with the theoretical analysis.

AMS subject classifications: 45D05, 45G10, 41A10, 65L60, 65L70

Key words: Nonlinear Volterra integral equations, domain decomposition method, Chebyshev–collocation spectral method, convergence analysis.

1 Introduction

Many problems arising from science, engineering and other fields lead to differential equations or integral equations. Sometimes a problem can be modeled by either differential equations or integral equations. Usually a differential equation (set) with boundary conditions can be correspondingly turned into integral equations, by which both dimensions of the problem considered and the numbers of nodes are reduced. As an advantage integral equations saves the cost of computing. However, in most of nonlinear cases it is difficult to get analytic solutions for integral equations. It is always important to develop numerical approximation techniques with easy-performance, high-accuracy and rapid-convergence. This is especially useful to integral equations. In recent decades, there are quite a few works on the numerical approaches of integral equations (see [1,2] and the references therein).

In recent years, spectral methods are being applied to integral equations. In [3], Elnagar and Kazemi investigated the Chebyshev spectral method for approximate solutions of the nonlinear Volterra-Hammerstein integral equations. In their treatment the

*Corresponding author. *Email addresses:* `hwu@staff.shu.edu.cn` (H. WU), `851747515@qq.com` (Y.-Z. Zhu), `18817612781@163.com` (H.-L. Wang), `xuhaishui@126.com` (L.-F. Xu)

integral term of the equation is dealt with by utilizing the Gaussian quadrature with the Chebyshev-Gauss-Lobatto points, and the integrand function has to be rewritten through being divided by the Chebyshev weight function in order to get the form with Chebyshev weight function. Their numerical experiments coincide with convergence, which demonstrates the applicability and the accuracy of the Chebyshev spectral method for integral equations, but no theoretical justification about the spectral rate of convergence was given in [3]. Tang and his collaborators have contributed a series of works to develop spectral methods for integral equations (see [4–10]). In [5], a Jacobi-collocation spectral method was applied to the Volterra integral equations of the second kind with a weakly singular kernel. The convergence was analyzed by means of the Lebesgue constants corresponding to the Lagrange interpolation polynomials, and polynomial approximation theory for orthogonal polynomials and operator theory. Their method provided the spectral rate of convergence, which was also demonstrated by their numerical results. In [6] the Legendre spectral collocation method was used to solve the Volterral integral equations with a smooth kernel function and a rigorous error analysis was given, in which exponentially decayed numerical errors can be obtained if the kernel function and the source function are sufficiently smooth.

The Chebyshev spectral collocation method is usually applied into integral equations with singular kernel [4, 5]. This is due to the fact that the Chebyshev spectral method is always accompanied with the weak singular weight function. For the spectral method of integral equations with smooth kernel function the Legendre method can be considered [6, 7]. Compared with the Legendre collocation method which has high stability but implicit expressions for collocation points and weights, the Chebyshev collocation method has explicit collection points and weights. In addition, the Chebyshev method can save computing time by means of the Fast Fourier Transfer method. Therefore it might be more convenient in practice and more popular in engineering calculation.

In recent years, along with the development of the technique of domain decomposition and parallel computing, more and more researchers and engineers begin to study domain decomposition spectral methods. The methods have been used in many fields. In the past the methods were mostly used in finding numerical solutions for partial differential equations (see [11] and the references therein). As for the domain decomposition method for the integral equation, one can refer to [10] and [12] for the recent decelopment. In [10], a parallel in time method to solve Volterra integral equations of second kind with smooth kernel function was proposed, which follows the spirit of the domain decomposition Legendre-Gauss spectral collocation method. A rigorous convergence analysis of the method was also provided in [10]. In [12], a multi-step Legendre-Gauss spectral collocation method for the nonlinear Volterra integral equations of the second kind was introduced. The authors also derived the optimal convergence of the hp-version of the method under the L^2-norm, which is confirmed by their numerical experiments.

In this paper, we extend the domain decomposition Chebyshev collocation spectral method to the second-kind Volterra integral equation. We consider the following Volterra