Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 5, pp. 810-826

DOI: 10.4208/aamm.2015.m904
October 2016

Identification of Elastic Orthotropic Material Parameters by the Singular Boundary Method

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Received 7 January 2015; Accepted (in revised version) 12 August 2015

Abstract. This article addresses the resolution of the inverse problem for the parameter identification in orthotropic materials with a number of measurements merely on the boundaries. The inverse problem is formulated as an optimization problem of a residual functional which evaluates the differences between the experimental and predicted displacements. The singular boundary method, an integration-free, mathematically simple and boundary-only meshless method, is employed to numerically determine the predicted displacements. The residual functional is minimized by the Levenberg-Marquardt method. Three numerical examples are carried out to illustrate the robustness, efficiency, and accuracy of the proposed scheme. In addition, different levels of noise are added into the boundary conditions to verify the stability of the present methodology.

AMS subject classifications: 62P30, 65M32, 65K05

Key words: Inverse problems, parameter identification, meshless method, singular boundary method, Levenberg-Marquardt method.

1 Introduction

Elastic constants are routinely obtained from laboratory tests on the specimens with a well-defined standardized geometry and loading. A uniaxial stress test is used to identify one or two material properties. Thus for orthotropic materials, such as wood, many crystals, and rolled metals, more mechanical tests along the orthogonal principal axes of the material are required to determine all the mechanical properties. Moreover, in some cases the mechanical property of materials may be changed due to the manufacturing

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conditions, and the test samples are not available [1–3]. As a consequence, the mixed numerical-experimental techniques are developed.

Thanks to the development of experimental devices and numerical methodologies in the past decades, the mixed numerical-experimental techniques have been successfully applied in the parameter identification problem [4–6]. Mixed numerical-experimental techniques, namely inverse methods, are based on static loading experiments, optimization techniques, full-field measurement techniques, and numerical modeling. The aim of the inverse methods is to identify the mechanical properties with a reduced experimental set-up. The advantage of the inverse methods lies in that non-standardized specimen geometries as well as more complicated boundary conditions and material models can be taken into consideration. For these reasons, the inverse methods are attractive to extract constitutive parameters from the stress/strain/displacement fields, either for composites or for other materials [7–9].

Quite a few numerical techniques are developed to solve the parameter identification problems, such as the constitutive equation gap method (CEGM) [10, 11], finite element method (FEM) [12–15], virtual fields method (VFM) [16–19], and boundary element method (BEM) [3, 20–22]. The basic idea of the VFM is to apply the principle of virtual work to the tested specimen with some explicit and independent virtual displacement fields. Each virtual field provides a linear equation based on the principle of virtual work and finally leads to a linear system to directly determine the unknown parameters. However, the specimen geometry must be defined to balance the influences of each unknown parameter and a set of available virtual fields leading to a well-conditioned system are required in the VFM. In the other methods, the parameter identification problem is formulated as an optimization problem of a least square residual functional which evaluates the differences between the experimental and predicted displacements. The numerical methods are only involved in the solution of the direct problems to obtain the predicted displacements with estimated parameters. The least square residual functional is minimized by an optimization method, i.e., Levenberg-Marquardt method in this study.

Among the numerical methods, the FEM requires mesh generation and re-meshing which is computationally expensive and sometimes mathematically troublesome, especially for the complex specimen. In addition, the FEM encounters the difficulties of raising the regularity of the approximation. The CEGM stems from a more general approach developed for updating FEM models from assessing quality of FEM meshes or vibration data. Up to now, another three techniques based on FEM models have been widely employed: the reciprocity gap method [23], the equilibrium gap method [24, 25], and multistep reciprocity gap function method [26]. A good survey of these techniques can be found in [27] with a focus on noisy data in [28]. By contrast, the BEM requires only meshing on the boundary. But nevertheless the BEM involves sophisticated mathematics and numerical integrations due to singular fundamental solutions. To avoid these difficulties, the method of fundamental solutions (MFS) [29, 30] approximates the solution with a linear combination of fundamental solutions with respect to the source points in