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Quasi-Static Linear Thermo-Viscoelastic Process with Irregular Viscous Dissipation

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Abstract. We consider a mathematical model which describes the quasi-static evolution of a thermo-viscoelastic linear body with taking into account the effects of internal forces which generate a non linear viscous dissipative function. We derive a variational formulation of the system of equilibrium equation and energy equation. An existence result of weak solutions was obtained in an appropriate function space.

AMS subject classifications: 35M10, 74D05, 74F05

Key words: Thermo-viscoelasticity, viscous dissipation.

1 Introduction

The constitutive laws with internal variables have been used in various publications in order to model the effect of internal variables in the behaviour of real bodies like metals, rocks polymers and so on, for which the rate of deformation depends on the internal variables. Some of the internal state variables considered by many authors are the spatial display of dislocation, the work-hardening of materials and the absolute temperature, see for examples and details [3,7,10,11,14,16,19] and references therein for other internal state variables.

The stress-strain behaviour has a direct relation with the temperature. Indeed, the work of internal forces generates a temperature and, inversely, variations of temperature may generate deformations in rigid materials. The physical experiences have prove that mechanical properties change dramatically with temperature, going from glass-like brittle behaviour at low temperatures to a rubber-like behaviour at high temperatures. In

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general, decreasing the strain rate has the same influence on the strain-strength characteristics as increasing the temperature: the material becomes softer and more ductile.

In the study of the influence of temperature on the behaviour of rigid materials (elastic or plastic) majority of mathematicians, see for example [14, 19], assumed, to simplify the calculations, that the dissipation is a Lipschitz function of strain, strain rate and stress tensors. But physically, this assumption can not describe all the forces of collision and interaction between the particles constituting the material. In fact, the dissipation function can be written as product of the stress tensor and the dissipative part of the strain rate tensor.

The aim object of this paper is to study the quasi-static evolution of linear thermo-viscoelastic materials by considering real viscous dissipative function. For this, we consider a rate-type constitutive law of the form

$$\frac{\partial\sigma}{\partial t}=\mathcal{A}\left(\boldsymbol{\varepsilon}\left(\frac{\partial\mathbf{u}}{\partial t}\right)\right)+\mu(\theta)\left(\mathcal{G}_1(\sigma)+\mathcal{G}_2(\varepsilon(\mathbf{u}))\right), \tag{1.1}$$

in which $\mathbf{u}$, σ represent, respectively, the displacement field and stress field, θ represents the absolute temperature, $\mathcal{A}$, $\mathcal{G}_1$ and $\mathcal{G}_2$ are real tensors and μ is a real function. The paper is organized as follows. In Section 2 we present the mechanical problem of the quasi-static evolution of thermo-viscoelastic materials, we introduce some notations and preliminaries and we derive the variational formulation of the problem. We prove in Section 3 the existence of solutions and we try to apply our results to the thermo-viscoelastic Maxwell model.

2 Problem statement and preliminaries

Let $\Omega\subset\mathbb{R}^n$ $(n=2,3)$ be a bounded domain with a Lipschitz boundary Γ, partitioned into two disjoint measurable parts Γ_1 and Γ_2 such that $\|\Gamma_1\|>0$ and let $Q=\Omega\times(0,T)$. We denote by $\mathbb{S}_n$ the space of symmetric tensors on $\mathbb{R}^n$. We define the inner product and the Euclidean norm on $\mathbb{R}^n$ and $\mathbb{S}_n$, respectively, by

$$\begin{aligned}
&\mathbf{u}\cdot\mathbf{v}=u_iv_i,\quad \forall\mathbf{u},\mathbf{v}\in\mathbb{R}^n \quad\text{and}\quad \boldsymbol{\sigma}\cdot\boldsymbol{\tau}=\sigma_{ij}\tau_{ij}\quad \forall\boldsymbol{\sigma},\boldsymbol{\tau}\in\mathbb{S}_n,\\
&\|\mathbf{u}\|=(\mathbf{u}\cdot\mathbf{u})^{\frac{1}{2}},\quad \forall\mathbf{u}\in\mathbb{R}^n\quad\text{and}\quad \|\sigma\|=(\sigma\cdot\sigma)^{\frac{1}{2}},\quad \forall\sigma\in\mathbb{S}_n.
\end{aligned}$$

Here and below, the indices i and j run from 1 to n and the summation convention over repeated indices is used.

For the rest of this article, we will denote by c possibly different positive constants depending only on the data of the problem. Denote by p' and q' the conjugates of p and