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Finite Element Simulations with Adaptively Moving Mesh for the Reaction Diffusion System

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Abstract. A moving mesh method is proposed for solving reaction-diffusion equations. The finite element method is used to solving the partial different equation system, and an efficient numerical scheme is applied to implement mesh moving. In the practical calculations, the moving mesh step and the problem equation solver are performed alternatively. Several numerical examples are presented, including the Gray-Scott, the Activator-Inhibitor and a case with a growing domain. It is illustrated numerically that the moving mesh methods costs much lower, compared with the numerical schemes on a fixed mesh. Even in the case of complex pattern dynamics described by the reaction-diffusion systems, the adapted meshes can capture the details successfully.

AMS subject classifications: 65M60, 65N50
Key words: moving mesh; finite elements; reaction-diffusion; growing domain; pattern formation.

1. Introduction

The pattern formation modeled with the reaction-diffusion systems are interested in various research fields, such as chemistry, biology, ecology and biochemistry [16–19]. It describes the interplay between several "species" when reactions among them are considered. The reaction terms are always nonequilibrium and nonlinear, and the diffusion effects are modeled with the Laplacian. Numerical simulation is one of the main strategies for practical investigations, see for e.g., [2, 6, 9, 11]. However, some numerical schemes may lead to plausible but non-physical solutions due to the presence of the nonlinearity [24]. High resolution numerical methods, including either high order schemes or sufficiently refined meshes, are capable of avoiding such miscalculations.

The adaptive numerical methods are commonly used in numerical simulations, which contain two main topics, the discretization method and the adaptive method.

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The Finite Difference Methods (FDM) and the Finite Element Methods (FEM) are both popular for discretization [10, 15], however, the FEM win a bit due to its better flexibility for complex topological geometries and the convenience in performing adaptive calculations. So that the FEM are preferred as the basic discretization method in this research. The mesh adaptivity is constantly used for better efficiency of the finite element calculations. The Adaptive Mesh Moving (AMM) and the Adaptive Mesh Refinement (AMR) are two different choices in the region of numerical simulations. While AMM plays an growing role in the last decade [7, 13, 14, 20, 24, 25]. In this research, the concerned small scale structure for pattern formation always appears locally in a moving space region and/or periodically in time, then we prefer the moving mesh strategy for the purpose of lower costs.

The fundamental idea of the AMM is to find a minimization of a so-called mesh-energy functional. Then different methodology for finding the minimization yield different moving mesh models. Huang [8] proposed the so-called Moving Mesh Partial Differential Equations (MMPDE) based on the gradient flow of the functional, and also provided several different types of MMPDEs, which is suitable for different type of physical models. Li *et al.* [12] suggest to find the minimization of the mesh-energy functional directly, and rigorous proof for the existence and convergence are given based the harmonic mapping theory. It is robust and effective when applied in different finite element applications [7, 22]. The corresponding implementation is also kindly shared by the authors under an open source policy. There are several further different variants for the moving mesh equations by other authors [4, 20]. In general, the linear systems arising from the discretization of the moving mesh models is required to be solved accurately enough for archiving the convergence. However, the efficiency is the mainly concerned issue for the numerical methods for mesh moving in this research, so that a simpler but efficient version moving mesh equation solver is applied.

Moving mesh finite element calculations are investigated for the reaction-diffusion system in this paper. The rest of this paper is organized as following. In the next section, the finite element scheme on a fixed mesh are introduced for the reaction-diffusion systems. In Section 3, we explain our numerical scheme for the moving mesh equations. Numerical examples are illustrated, and comparisons are made between with and without the moving mesh approach. Some observation and conclusions are made at the last section.

2. The finite element formulations of model problem

2.1. The reaction-diffusion systems

There are different types of classical reaction-diffusion systems in the pattern formation theory: Brusselator, Gray-Scott, Activator-Inhibitor and Schnakenberg etc., which has only slight differences between them on some parameters. Considering two different species living in two dimensional fixed domain Ω, let scalar functions $u(t, x, y)$ and $v(t, x, y)$ be their densities respectively, where t is the temporal variable and x, y are