Numer. Math. Theor. Meth. Appl. Vol. **11**, No. 4, pp. 795-809
doi: 10.4208/nmtma.2018.s07 November 2018

A Numerical Study of Fluid-Particle Interaction with Slip Boundary Condition

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Received 30 August 2017; Accepted (in revised version) 19 December 2017

Dedicated to Professor Xiaoqing Jin on the occasion of his 60th birthday

Abstract. In this paper, we present a numerical study of the effect of slip in the fluid-particle interaction. The motion of the particle is described by the Newton's second law and the flows are simulated by solving the incompressible Navier-Stokes equations with the Navier slip boundary condition. Numerical schemes are designed using the extended finite element method (XFEM) combined with the temporary arbitrary Lagrangian-Eulerian (tALE) technique. In this method, both the fluid dynamics and the motion of particle are efficiently computed on a fixed Cartesian mesh. With the XFEM, the discontinuities at the particle boundary are naturally captured by the Heaviside-enriched finite element basis functions. With the tALE technique, field variables at the previous time level are mapped onto the computational mesh at the current time level, hence regeneration or deformation of meshes can be avoided. To study the effect of the slip, we simulate the rotation of an ellipsoidal particle in a simple shear flow and compare with the analytic results from the theory of Jeffery orbit.

AMS subject classifications: 65M10, 78A48

Key words: Fluid-particle interaction, direct numerical simulation, extended finite element method, temporary arbitrary Lagrangian-Eulerian, Jeffery orbit.

1. Introduction

Considerable attentions have been paid for both experimental and numerical investigations of fluid-particle interaction problems. Numerical studies of fluid-particle systems concentrate on the description of interface, field variable discontinuity, and hydrodynamic interaction between fluid and solid objects. In the past decade, two categories of numerical simulations have been developed, including the continuum approach and the direct numerical simulation (DNS) approach. In the continuum approach, solid particles and

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fluids are viewed as interpenetrating mixtures with different viscosities that governed by conservation laws [21, 27, 29, 30]. Despite its efficiency and flexibility, the continuum approach suffers from false response from the inside viscous material which is used to mimic the rigid objects, causing inaccurate hydrodynamical effects. On the other hand, the DNS approach, where the fluid is described by Navier-Stokes equations and the motion of rigid-body is governed by the Newton's second law, provides a fundamental understanding of the mechanism and more details of fluid-particle interactions.

A classical method in DNS for fluid-particle interaction problems is to use the boundary-fitted mesh by aligning the solid boundary with element edges [18–20]. In this approach, the governing equations are solved only in the fluid domain and the discontinuous characteristics are naturally captured on the boundaries of particles. To handle moving particles, this approach incorporates the ALE technique where mesh nodes near particles follow the motion of particles in a Lagrangian manner, while mesh nodes far away from particles are remained stationary in an Eulerian manner. The creation of new unstructured meshes is often needed for the region occupied by the fluid. In general, it is very time consuming to generate boundary-fitted meshes if complex geometries are involved.

An alternative approach is the fictitious domain method developed by Glowinski et al. [11–13]. The basic idea of this approach is to extend the problem in a geometrically complex domain to a simple and regular domain by generalizing the weak formulation of flows from the fluid domain to the fictitious domain that represents the particles. This approach enforces the constraint of rigid-body motion with a distributed Lagrange multiplier to introduce an additional body force to the interior of the region of particles. Field variables at the interface are required to be interpolated between the values of the physical fluid and the fictitious fluid.

Recently, a novel numerical method, the extended finite element method, or XFEM, was developed to capture arbitrary discontinuous at the interface on a fixed Cartesian mesh. The XFEM generalizes the standard Galerkin finite element method with additional degrees of freedom to handle discontinuities. The methodology was first developed by Moës et al. [25] to solve crack problems. It has been extended to several other problems including elastic problems with holes [26], fluid-structure interaction [10] and particulate flow problems [8]. Gerstenberger and Wall [10] utilized the XFEM for problems of fluid-structure interaction (FSI) using a Heaviside function to define the discontinuities. Choi et al. [8] presented an application of XFEM for the simulation of moving particles in a viscoelastic fluid, with a no-slip boundary condition imposed on the particle boundary through Lagrange multipliers. In their work, a temporary arbitrary Lagrangian-Eulerian (tALE) scheme was proposed to handle the motion of particles by mapping the time dependent variables along with the ALE meshes between two time levels.

In [30], Zhang et al. investigated the effects of boundary slip on the orientational motion of an anisotropic particle in a simple shear flow. A fluid-particle system is numerically solved using the fluid particle dynamics (FPD) method [27]. The dependence of the cross-coupling coefficient on the slip length at particle surface is measured, showing that the boundary slip can enhance the effective anisotropy of the particle and hence the cross coupling between the rotational torque and the shear stress.