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NUMERICAL APPROXIMATION OF VISCOELASTIC FLUID-STRUCTURE INTERACTION PROBLEMS

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Dedicated to Professor William J. Layton on the occasion of his 60^{th} *birthday*

Abstract. We consider a fluid-structure interaction (FSI) problem that consists of a viscoelastic fluid flow and a linear elastic structure. The system is formulated as (i) a monolithic problem, where the matching conditions at the moving interface are satisfied implicitly, and (ii) a partitioned problem, where the fluid and structure subproblems are coupled by Robin-type boundary conditions along the interface. Numerical algorithms are designed based on the Arbitrary Lagrangian-Eulerian (ALE) formulation for the time-dependent fluid domain. We perform numerical experiments to compare monolithic and partitioned schemes and study the effect of stress boundary conditions on the inflow portion of moving interface.

Key words. Fluid-structure interaction, viscoelastic fluid.

1. Introduction

Fluid-structure interaction (FSI) problems are multi-physics problems involving fluid flows and deformable structures. Such problems are widely used in engineering and biological applications where a surrounding or internal fluid interacts with a movable structure. In general, such FSI problems are solved by either a monolithic or a partitioned approach. The monolithic approach solves the entire problem within one complex system considering fluid and structure together while treating the interface conditions implicitly. The partitioned approach, on the other hand, decouples the interaction system into two subproblems and uses local solvers. The interface-matching conditions are explicit as a bridge of the two subproblems. A monolithic algorithm requires a large memory storage and a special solver, but avoids the stability issue existing in many partitioned algorithms. When densities of the fluid and structure are close, explicit staggered approaches often fail and even implicit staggered methods may become unstable due to the added mass effect [8, 13, 16, 18, 30].

Much work has been done with both monolithic and partitioned approaches for Newtonian FSI problems where the fluid viscosity is constant. Fluids of interest in some FSI problems include blood, air in the bronchial passages, paint, latex, and other industrial polymers. There are some difficulties of simulating a fluid which is non-Newtonian in nature since the shear stress of fluid is not proportional to the shear rate. In simulations of blood flow, Newtonian models have been used and have performed well in most cases where a larger vessel is considered such as the aorta. However, it is well known that blood flow through small caliber vessels shows non-Newtonian behavior, therefore demanding a more accurate and realistic model [2, 4, 5, 15, 19, 21, 25, 28, 29].

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Unlike Newtonian FSI, which have been thoroughly elucidated [6, 7, 11, 12], few analytical and numerical studies have been undertaken for interaction of non-Newtonian fluids and elastic solids. Shear-thinning viscoelastic fluid models are presented and numerically tested for a blood flow in [3, 5]. Chan et al. considered Carreau fluid and power law fluid for FSI problems and compared the numerical results of the two different fluid models [9]. An energy estimate and numerical results using a splitting method is presented by J.Janela et al. for a generalized Newtonian shear-thinning FSI problem [19], and an extended study was performed by the same authors for several absorbing boundary conditions [20]. Relevant numerical studies considering a viscoelastic flow through a flexible channel were done by Chen et al. [10], and a mass-spring-dashpot prototype model was also examined by the same authors. In our recent work [23, 24], we have analyzed quasi-Newtonian FSI problems for stability and finite element error estimates.

In this paper, we consider a viscoelastic fluid where a separate hyperbolic differential constitutive equation is required to describe the complicated stress-deformation relation. Difficulties arise from both analytical and computational aspects due to the hyperbolic character and the lack of a stabilizing term for the stress. It is well known that for a viscoelastic fluid, a stress boundary condition on the inflow boundary must be imposed to ensure the well-posedness of the model equations. When a partitioned scheme is considered for simulating viscoelastic FSI, an extra difficulty is encountered due to (i) the movement of inlet and outlet boundaries along the interface of two substructures and (ii) the lack of information on the stress along the moving boundary. There are a few studies on viscoelastic FSI problems by partitioned methods in the literature [3, 5, 10]; however, no numerical methods or discussion have been reported to handle stress boundary conditions on the interface. In this work, we simulate the viscoelastic FSI problem using a monolithic and a partitioned algorithm and investigate how the stress boundary condition affects the FSI system.

The paper is organized as follows. Section 2 introduces model equations of the fluid-structure system with initial and boundary conditions. The matching conditions for the two dynamics on the interface are also provided in this section. In Section 3, we provide a monolithic and a partitioned formulation in the ALE framework. The last section presents numerical experiments and comparison of numerical results by algorithms discussed in Section 3.

2. Models Equations and Framework

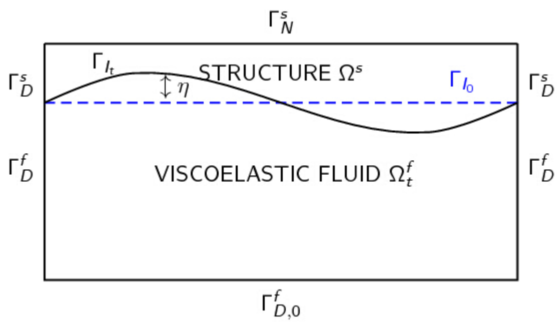

FIGURE 1. Fluid-Structure interaction domain.