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On Weighted L^p- Approximation by Weighted Bernstein-Durrmeyer Operators

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Abstract. In the present paper, we establish direct and converse theorems for weighted Bernstein-Durrmeyer operators under weighted L^p-norm with Jacobi weight $w(x)=x^\alpha(1-x)^\beta$. All the results involved have no restriction $\alpha,\beta<1-\frac{1}{p}$, which indicates that the weighted Bernstein-Durrmeyer operators have some better approximation properties than the usual Bernstein-Durrmeyer operators.

Key Words: Weighted L^p-approximation, weighted Bernstein-Durrmeyer operators, direct and converse theorems.

AMS Subject Classifications: 41A10, 41A25

1 Introduction

Let

$$w(x)=x^\alpha(1-x)^\beta,\quad \alpha,\beta>-1,\quad 0\le x\le 1,$$

be the classical Jacobi weights. Let

$$L_w^p:=\begin{cases}\{f: wf\in L^p(0,1)\}, & 1\le p<\infty,\\ \Big\{f: f\in C(0,1),\ \lim_{x(1-x)\to 0}(wf)(x)=0\Big\}, & p=\infty.\end{cases}$$

Set

$$\|f\|_{p,w,I}=\begin{cases}\left(\int_I |(wf)(x)|^p dx\right)^{1/p}, & 1\le p<\infty,\\ \sup_{x\in I}|(wf)(x)|, & p=\infty.\end{cases}$$

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When $I=[0,1]$, we briefly write $\|f\|_{p,w}$ instead of $\|f\|_{p,w,[0,1]}$. Obviously, $\|f\|_{p,w}$ is the norm of L^p_w spaces.

For any $f\in L^p([0,1])$, $1\le p\le\infty$, the corresponding Bernstein-Durrmeyer operators $M_n(f,x)$ are defined as follows:

$$M_n(f,x)=(n+1)\sum_{k=0}^{n}p_{n,k}(x)\int_0^1 p_{n,k}(t)f(t)dt,\quad x\in[0,1],$$

where

$$p_{n,k}(x)=\left(\begin{array}{c}k\\ n\end{array}\right)x^k(1-x)^{n-k},\quad x\in[0,1],\quad k=0,1,\cdots,n.$$

The approximation properties of $M_n(f,x)$ in L^p_w were also studied by Zhang (see [9]). Some approximation results were given under the restrictions

$$-\frac{1}{p}<\alpha,\beta<1-\frac{1}{p}$$

on the weight parameters. Generally speaking, the restrictions can not be eliminated for the approximation by $M_n(f,x)$. For the weighted approximation by Kantorovich-Bernstein operators defined by

$$K_n(f,x):=\sum_{k=0}^{n}(n+1)\int_{\frac{k}{n+1}}^{\frac{k+1}{n+1}}f(u)du\,p_{nk}(x),$$

the situation is similar (see [5]). Recently, Della Vecchia, Mastroianni and Szabados (see [2]) introduced a weighted generalization of the $K_n(f,x)$ as follows:

$$K_n^{\#}(f,x):=\sum_{k=0}^{n}\frac{\int_{I_k}(wf)(t)dt}{\int_{I_k}w(t)dt}p_{nk}(x),\quad x\in[0,1]. \tag{1.1}$$

When $\alpha=\beta=0$, $K_n^{\#}(f,x)$ reduces to the classical Kantorovich-Bernstein operator $K_n(f,x)$. Della Vecchia, Mastroianni and Szabados obtained the direct and converse theorems and a Voronovskaya-type relation in [2], and solved the saturation problem of the operator in [3]. Their results showed that $K_n^{\#}(f,x)$ allows a wider class of functions than the operator $K_n(f,x)$. In fact, they dropped the restrictions $\alpha,\beta<1-\frac{1}{p}$ on the weight parameters. Later, Yu (see [8]) introduced another kind of modified Bernstein-Kantorvich operators, and established direct and converse results on weighted approximation which also have no restrictions $\alpha,\beta<1-\frac{1}{p}$.

Then, a natural question is: can we modified the Bernstein-Durrmeyer operators properly such that the restrictions $\alpha,\beta<1-\frac{1}{p}$ on weighted approximation can be dropped? In the present paper, we will show that the weighted Bernstein-Durrmeyer operator