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SECOND-ORDER TWO-SCALE ANALYSIS METHOD FOR DYNAMIC THERMO-MECHANICAL PROBLEMS OF COMPOSITE STRUCTURES WITH CYLINDRICAL PERIODICITY

HAO DONG[1,*], YUFENG NIE[2], JUNZHI CUI[3], ZIHAO YANG[2], AND ZIQIANG WANG[4]

Abstract. In this paper, a novel second-order two-scale (SOTS) analysis method and corresponding numerical algorithm is developed for dynamic thermo-mechanical problems of composite structures with cylindrical periodicity. The formal SOTS solutions are successfully constructed by the multiscale asymptotic analysis. Then we theoretically explain the necessity of developing the SOTS solutions by the error analysis in the pointwise sense. Futhermore, the convergence result with an explicit rate for the SOTS solutions is obtained. In addition, a SOTS numerical algorithm is presented to effectively solve these multiscale problems. Finally, some numerical examples verify the feasibility and validity of the SOTS numerical algorithm we proposed. This study offers a unified multiscale framework that enables the simulation and analysis of thermo-mechanical coupled behavior of composite structures with cylindrical periodicity.

Key words. Dynamic thermo-mechanical problem, multiscale asymptotic analysis, composite structure, cylindrical periodicity, SOTS numerical algorithm.

1. Introduction

In the past decades, composite materials have been widely used in engineering applications owing to their attractive physical and mechanical properties. With the appearance of various complex and extreme environments, composite materials usually served under multi-physics coupled circumstances, such as electro-mechanical, thermo-electrical, thermo-mechanical and magneto-electro-thermo-elastic, etc. Due to a great application prospect, the thermo-mechanical performances of composite materials have been a research hotspot of scientists and engineers. To the best of our knowledge, some studies have performed on dynamic thermo-mechanical problems of composites. However, most of these studies focused on one-way thermo-mechanical coupled problems [1–5], namely only the thermal effects affect the mechanical field. Besides, some researchers devoted to the two-way thermo-mechanical coupled problems which are fully coupled hyperbolic and parabolic systems, but their researches were based on the cartesian coordinate system [6–10]. To the best of our knowledge, the structures made of the composites with cylindrical periodic configurations have a great application value in practical engineering, such as composite shells, composite cylinder, composite tube, etc. In recent years, some research results for composite structures with cylindrical periodicity have appeared [5, 11–15]. However, up to now there is a lack of adequate research on dynamic thermo-mechanical problems of composite structures with cylindrical periodicity.

The subject of this paper is to develop a SOTS analysis method and associated numerical algorithm for dynamic thermo-mechanical problems of composite

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*Corresponding author.

structures with cylindrical periodicity. In such cases, the direct numerical computation of these multiscale problems needs a tremendous amount of computational resources to capture the micro-scale behaviors due to large heterogeneities (caused by inclusions or holes) in composite structures. Furthermore, the stability of numerical scheme for these coupled systems with cylindrical periodic configurations is also a difficult problem to handle. From the point of view of theoretical analysis, the error estimate of SOTS solutions with an explicit convergence rate is hard to gain due to lack of a prior estimate for wave equations with nonhomogeneous boundary condition. In order to deal with these difficulties, we develop a SOTS method to overcome numerical difficulties based on asymptotic homogenization method (AHM), finite element method (FEM) and finite difference method (FDM). On the other hand, we impose the homogeneous Dirichlet condition on auxiliary cell problems. At this case, the explicit convergence rate of SOTS solutions is easily obtained because the SOTS solutions will satisfy automatically the boundary condition of governing equations under some assumptions.

This paper is organized as follows. In Sections 2, the detailed construction of the SOTS solutions for dynamic thermo-mechanical problems of composite structures with cylindrical periodicity is given by multiscale asymptotic analysis. Moreover, the error analysis in the pointwise sense of first-order two-scale (FOTS) solutions and SOTS solutions is obtained, respectively. Through the above analysis, we theoretically explain the importance of developing the SOTS solutions in capturing micro-scale information. In Section 3, an explicit convergence rate for the SOTS solutions are derived under some hypotheses. In Section 4, a SOTS numerical algorithm based on FEM and FDM is presented to solve these multiscale problems effectively. In Section 5, some numerical results are given to verify the feasibility and validity of our SOTS algorithm. Finally, some conclusions are given in Section 6.

For convenience, we use the Einstein summation convention on repeated indices in this paper. Besides, the notation δ_{ij} is the Kronecker symbol, and if $i = j$, $\delta_{ij} = 1$, or $\delta_{ij} = 0$.

2. The multiscale asymptotic analysis of governing equations

Consider governing equations for dynamic thermo-mechanical problems of composite structures with cylindrical periodicity as follows

$$(1)\quad \begin{cases} \rho^\varepsilon \dfrac{\partial^2 u_r^\varepsilon}{\partial t^2} - \Big(\dfrac{\partial \sigma_{rr}^\varepsilon}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_{r\theta}^\varepsilon}{\partial \theta} + \dfrac{\partial \sigma_{rz}^\varepsilon}{\partial z} + \dfrac{\sigma_{rr}^\varepsilon - \sigma_{\theta\theta}^\varepsilon}{r}\Big) = f_r \ \text{ in } \ \Omega \times (0,T], \\ \rho^\varepsilon \dfrac{\partial^2 u_\theta^\varepsilon}{\partial t^2} - \Big(\dfrac{\partial \sigma_{r\theta}^\varepsilon}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_{\theta\theta}^\varepsilon}{\partial \theta} + \dfrac{\partial \sigma_{z\theta}^\varepsilon}{\partial z} + 2\dfrac{\sigma_{r\theta}^\varepsilon}{r}\Big) = f_\theta \ \text{ in } \ \Omega \times (0,T], \\ \rho^\varepsilon \dfrac{\partial^2 u_z^\varepsilon}{\partial t^2} - \Big(\dfrac{\partial \sigma_{zr}^\varepsilon}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_{z\theta}^\varepsilon}{\partial \theta} + \dfrac{\partial \sigma_{zz}^\varepsilon}{\partial z} + \dfrac{\sigma_{zr}^\varepsilon}{r}\Big) = f_z \ \text{ in } \ \Omega \times (0,T], \\ \rho^\varepsilon c^\varepsilon \dfrac{\partial T^\varepsilon}{\partial t} + \Big(\dfrac{\partial q_r^\varepsilon}{\partial r} + \dfrac{1}{r}\dfrac{\partial q_\theta^\varepsilon}{\partial \theta} + \dfrac{\partial q_z^\varepsilon}{\partial z} + \dfrac{q_r^\varepsilon}{r}\Big) + \widetilde{T}\beta_{ij}^\varepsilon \dfrac{\partial \varepsilon_{ij}^\varepsilon}{\partial t} = h \ \text{ in } \ \Omega \times (0,T], \\ \mathbf{u}^\varepsilon(\mathbf{x},t) = \widehat{\mathbf{u}}(\mathbf{x},t), \ \ T^\varepsilon(\mathbf{x},t) = \widehat{T}(\mathbf{x},t) \ \text{ on } \ \partial\Omega \times (0,T], \\ \mathbf{u}^\varepsilon(\mathbf{x},0) = \mathbf{u}^0, \ \ \dfrac{\partial \mathbf{u}^\varepsilon(\mathbf{x},t)}{\partial t}\Big|_{t=0} = \mathbf{u}^1(\mathbf{x}), \ \ T^\varepsilon(\mathbf{x},0) = \widetilde{T} \ \text{ in } \ \Omega. \end{cases}$$

where Ω is a bounded convex domain $(0 < r < \infty)$ in $\mathbb{R}^3$ with a boundary $\partial\Omega$; The u_r^ε, u_θ^ε, u_z^ε and T^ε in (1) are undetermined displacement and temperature fields; $\widehat{\mathbf{u}}(\mathbf{x},t)$, $\widehat{T}(\mathbf{x},t)$ and $\mathbf{u}^1(\mathbf{x})$ are known functions with macro-coordinates $\mathbf{x} = (r,\theta,z)$; ε represents the characteristic periodic unit cell size; ρ^ε and c^ε are the mass density