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A Reformulated Convex and Selective Variational Image Segmentation Model and its Fast Multilevel Algorithm

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Abstract. Selective image segmentation is the task of extracting one object of interest among many others in an image based on minimal user input. Two-phase segmentation models cannot guarantee to locate this object, while multiphase models are more likely to classify this object with another features in the image. Several selective models were proposed recently and they would find local minimizers (sensitive to initialization) because non-convex minimization functionals are involved. Recently, Spencer-Chen (CMS 2015) has successfully proposed a convex selective variational image segmentation model (named CDSS), allowing a global minimizer to be found independently of initialization. However, their algorithm is sensitive to the regularization parameter μ and the area parameter θ due to nonlinearity in the functional and additionally it is only effective for images of moderate size. In order to process images of large size associated with high resolution, urgent need exists in developing fast iterative solvers. In this paper, a stabilized variant of CDSS model through primal-dual formulation is proposed and an optimization based multilevel algorithm for the new model is introduced. Numerical results show that the new model is less sensitive to parameter μ and θ compared to the original CDSS model and the multilevel algorithm produces quality segmentation in optimal computational time.

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Key words: Active contours, image segmentation, level sets, multilevel, optimization methods, energy minimization.

1. Introduction

Image segmentation is a fundamental task in image processing aiming to obtain meaningful partitions of an input image into a finite number of disjoint homogeneous regions. Segmentation models can be classified into two categories, namely, edge based and region based models; other models may mix these categories. Edge based models refer to

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the models that are able to drive the contours towards image edges by influence of an edge detector function. The snake algorithm proposed by Kass et al. [28] was the first edge based variational model for image segmentation. Further improvement on the algorithm with geodesic active contours and the level-set formulation led to effective models [13, 40]. Region-based segmentation techniques try to separate all pixels of an object from its background pixels based on the intensity and hence find image edges between regions satisfying different homogeneity criteria. Examples of region-based techniques are region growing [8,26], watershed algorithm [9,26], thresholding [26,44], and fuzzy clustering [41]. The most celebrated (region-based) variational model for the images (with and without noise) is the Mumford-Shah [35] model, reconstructing the segmented image as a piecewise smooth intensity function. Since the model cannot be implemented directly and easily, the Mumford-Shah general model [35] was often approximated. The Chan-Vese (CV) [20] model is simplified and reduced from [35], without approximation. The simplification is to replace the piecewise smooth function by a piecewise constant function (of two constants c_1, c_2 or more) and, in the case of two phases, the piecewise constant function divides an image into the foreground and the background. A new variant of the CV model [20] has been proposed by [7] by taking the Euler's elastica as the regularization of segmentation contour that can yield to convex contours. Another interesting model named second order Mumford-Shah total generalized variation was developed by [23] for simultaneously performs image denoising and segmentation.

The segmentation models described above are for global segmentation due to the fact that all features or objects in an image are to be segmented (though identifying all objects is not guaranteed due to non-convexity). Selective image segmentation aims to extract one object of interest in an image based on some additional information of geometric constraints [24, 39, 43]. This task cannot be achieved by global segmentation. Some effective models are Badshah-Chen [6] and Rada-Chen [39] which used a mixed edge based and region based ideas, and area constraints. Both models are non-convex. A non-convex selective variational image segmentation model, though effective in capturing a local minimiser, is sensitive to initialisation where the segmentation result relies heavily on user input.

While the above selective segmentation models are formulated based on geometric constraints in [24, 25], there are another way of defining the geometric constraints that can be found in [33] where geometric points outside and inside a targeted object are given. Their model make use the Split Bregman method to speed up convergence. Although our paper based on geometric constraint defining in [24,25], later, we shall compare our work with [33].We called their model as NCZZ model.

In 2015, Spencer-Chen [42, 43] has successfully designed a Convex Distance Selective Segmentation model (named as CDSS). This variational model allows a global minimizer to be found independently of initialization, given knowledge of c_1, c_2. The CDSS model [43] is challenging to solve due to its penalty function $\nu(u)$ being highly nonlinear. Consequently, the standard addition operator splitting method (AOS) is not adequate. An enhanced version of the AOS scheme was proposed in [43] by taking the approximation of $\nu'(u)$ which based on its linear part [42,43]. Another factor that affects the [43] model