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Bifurcation in a Differential-Algebra Predator-Prey System with Time Lag Effects

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Abstract. A predator-prey system with Holling type II functional response and a time lag is described by a delayed differential-algebra system and the local asymptotic stability and Hopf bifurcation of such system is studied. It is shown that if the time lag increases, a sequence of Hopf bifurcations can occur. The stability and direction of the Hopf bifurcations are studied by using center manifold theory for functional differential equations. A numerical example illustrates our theoretical findings.

AMS subject classifications: 92D25

Key words: Predator-prey, bifurcation, time lag, parametrisation, harvesting.

1. Introduction

The interactions between predators and preys are arguably the building blocks of complex ecosystems [37]. Therefore, since the last century the dynamical behaviors of predator-prey systems attracted extensive attention in theoretical ecology and applied mathematics. The functional response is a significant factor in population dynamics. It shows the average number of prey per time unit for an individual predator. The simplest functional response $mx(t)$, where $m > 0$ is the capture rate of the predator species, is a linear function of the prey density $x(t)$, introduced in the celebrated Lotka-Volterra predator-prey system [37]. It is clear that the linear functional response is unbounded and does not produce saturation. Later on, a more reasonable Holling type II functional response of the form of $mx(t)/(a + x(t))$ with the so-called half capturing saturation constant $a > 0$ has been considered [19, 20]. The Holling type II functional response is bounded. It indicates that the amount of prey consumed by a predator per unit of time is still finite, even if the prey density $x(t)$ is very large. In the present paper, we consider the basic predator-prey system of the Holling type II — viz.

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$$\begin{aligned} \dot{x}(t) &= x(t)\left(r - \frac{r}{K}x(t) - \frac{my(t)}{a+x(t)}\right), \\ \dot{y}(t) &= y(t)\left(-d + \frac{cmx(t)}{a+x(t)}\right), \end{aligned} \tag{1.1}$$

where $x(t)$ and $y(t)$ denote, respectively, the population densities of prey and predators at the time t. The positive constants r, K, d and c represent, respectively, the intrinsic growth rate of prey population in absence of predators, the carrying capacity of the environment, the death rate of predator species and the conversion rate of predator population (converting the captured prey into predators).

It was pointed out by Kuang [26] that any population dynamics of a predator-prey system without time lag is an approximation at best, since the time lag reflects the past history of the system and can affect the present and future dynamical behaviors. On the other hand, compared to ordinary differential systems, the differential systems with time lag usually exhibit much richer and more complex dynamics such as stability switches, multistability, Takens-Bogdanov bifurcation, oscillations and periodic motions — cf. Refs. [14, 26, 29, 33, 40, 42, 43, 45, 49, 50]. Following May [33], we add a time lag τ to the specific prey growth term $r-(r/K)x(t)$ in the system (1.1). Here, τ can be considered as the gestation period of prey species. Consequently, the predator-prey system with a time lag takes the form

$$\begin{aligned} \dot{x}(t) &= x(t)\left(r - \frac{r}{K}x(t-\tau) - \frac{my(t)}{a+x(t)}\right), \\ \dot{y}(t) &= y(t)\left(-d + \frac{cmx(t)}{a+x(t)}\right). \end{aligned} \tag{1.2}$$

Let us note that harvesting on biological populations are often practiced in real life, and the catches are commonly sold in the market for economic benefits. Therefore, here we consider a system with prey harvesting and investigate the economic benefits of the harvesting by using an economic equation in Ref. [15] — viz.

$$\text{Net Economic Revenue} = \text{Total Revenue} - \text{Total Cost}. \tag{1.3}$$

Let $E(t)$ denote the harvesting effort on the prey species, $\mathbb{P}(t)$ the unit selling price of the catch and $\mathbb{C}(t)$ the unit harvesting cost. Then $E(t)x(t)$ is the number of catches. We assume that the market has a constant demand for the catches, so that they always can be sold out here. Note that the selling price $\mathbb{P}(t)$ is a decreasing function of the market supply $E(t)x(t)$ [12, 31], and the harvesting cost $\mathbb{C}(t)$ varies inversely to the population density $x(t)$. Consequently, we take $\mathbb{P}(t)$ and $\mathbb{C}(t)$ as $b/(c+E(t)x(t))$ and $e/x(t)$ respectively, where b, c and e are positive constants, b/c is the maximum unit selling price and e the harvesting cost for the unit population density of prey species. Thus, taking into account the economic equation (1.3), we have

$$\begin{aligned} \text{Total Revenue} &= \mathbb{P}(t)E(t)x(t) = \frac{bE(t)x(t)}{c+E(t)x(t)}, \\ \text{Total Cost} &= \mathbb{C}(t)E(t)x(t) = eE(t). \end{aligned}$$