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On the Uniqueness of Traveling Forced Curvature Fronts in a Fibered Medium

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Abstract. We investigate traveling fronts, including pulsating ones, of a forced curvature flow in a plane fibered medium. The main topic of this note is an uniqueness issue of such traveling fronts. In addition to line-shaped profiles, we also consider traveling fronts in the form of V-shaped parabolas.

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1 Introduction

In this note, we will be interested in traveling fronts of a forced curvature flow equation

$$V_n = R + K \tag{1.1}$$

in the plane containing periodic striations. V_n is the normal velocity of a propagating interface $\Gamma(t)$, K is its mean curvature and R is the driving force. For example if Γ is a flame front, then R corresponds to the combustion rate of the burning material. In all cases, we will suppose that the function R is smooth and verifies

$$0 < R_m \leq R \leq R_M. \tag{1.2}$$

Before going further, let us give a definition of a traveling front of Eq. (1.1).

Definition 1.1. $\Gamma(t)$, solution of (1.1) will be called a traveling front if there exists a constant vector $\mathbf{v} \in \mathbb{R}^2$ such that

$$\Gamma(t) = \Gamma_0 + \mathbf{v}\, t$$

for all $t \in \mathbb{R}$. Then Γ_0 is the (constant) profile of the traveling front and $|\mathbf{v}|$, its speed, see Figure 1.

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Note that if $\Gamma(t)$ can be represented by the graph of a function u in the x-y plane, for example

$$\Gamma(t)=\{(x,y)/y=u(x,t)\},$$

then V_n is given by

$$V_n=\frac{u_t}{\sqrt{1+u_x^2}},$$

so that Equation (1.1) becomes

$$u_t-R\sqrt{1+u_x^2}=\frac{u_{xx}}{1+u_x^2}, \quad t\in\mathbb{R}, x\in\mathbb{R}. \tag{1.3}$$

Now if $\Gamma(t)$ is a traveling front in the plane, we can suppose without loss of generality that $\mathbf{v}$ is parallel to the y-axis *i.e.* $\mathbf{v}={}^t(0,c)$. Then $u(x,t)$ will be given by

$$u(x,t)=c\,t+\varphi(x),$$

so that Equation (1.3) becomes

$$c-R\sqrt{1+\varphi_x^2}=\frac{\varphi_{xx}}{1+\varphi_x^2}, \quad x\in\mathbb{R}. \tag{1.4}$$

In the above, c is the speed and φ the constant profile of the wave. The pair (c,φ) will be called a traveling wave solution (*TWS*) of Eq. (1.3). Note that every solution φ of (1.4) is defined up to an additive constant.

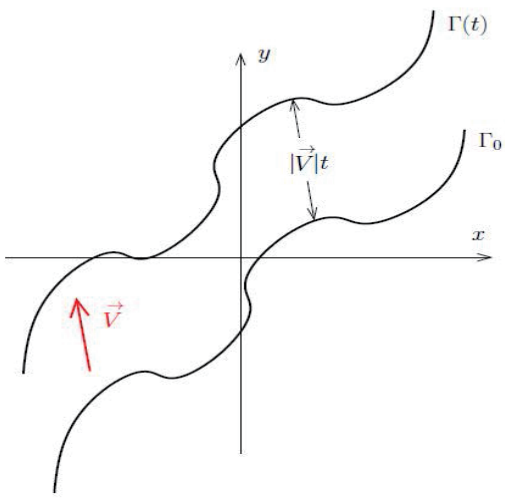

Figure 1: A TWS: a constant profile moving with a constant speed in some given direction.