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The Plasmonic Resonances of a Bowtie Antenna

Eric Bonnetier[1,*], Charles Dapogny[2], Faouzi Triki[2] and Hai Zhang[3]

[1] *Institut Fourier, Université Grenoble-Alpes, CS 40700, 38058 Grenoble Cedex 09, France*
[2] *Laboratoire Jean Kuntzmann, Université Grenoble-Alpes, 700 Avenue Centrale, 38401 Domaine Universitaire de Saint-Martin-d'Hères, France*
[3] *Department of Mathematics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong*

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Abstract. Metallic bowtie-shaped nanostructures are very interesting objects in optics, due to their capability of localizing and enhancing electromagnetic fields in the vicinity of their central neck. In this article, we investigate the electrostatic plasmonic resonances of two-dimensional bowtie-shaped domains by looking at the spectrum of their Poincaré variational operator. In particular, we show that the latter only consists of essential spectrum and fills the whole interval $[0,1]$. This behavior is very different from what occurs in the counterpart situation of a bowtie domain with only close-to-touching wings, a case where the essential spectrum of the Poincaré variational operator is reduced to an interval σ_{ess} strictly containing in $[0,1]$. We provide an explanation for this difference by showing that the spectrum of the Poincaré variational operator of bowtie-shaped domains with close-to-touching wings has eigenvalues which densify and eventually fill the remaining parts of $[0,1]\setminus\sigma_{ess}$ as the distance between the two wings tends to zero.

Key Words: Neumann-Poincaré operator, corner singularity, spectrum, resonance.

AMS Subject Classifications: 35P05, 45B05

1 Introduction

Surface plasmons are strongly localized electromagnetic fields that result from electron oscillations on the surface of metallic particles. Typically, this resonant behavior occurs when the real parts of the dielectric coefficients of the particles are negative and when their size is comparable to or smaller than the wavelength of the excitation. For instance,

*Corresponding author. *Email addresses:* `Eric.Bonnetier@univ-grenoble-alpes.fr` (E. Bonnetier), `Charles.Dapogny@univ-grenoble-alpes.fr` (Ch. Dapogny), `Faouzi.Triki@univ-grenoble-alpes.fr` (F. Triki), `haizhang@ust.hk` (H. Zhang)

this is the case of gold or silver nanoparticles, 20-50nm in diameter, when they are illuminated in the frequency range of visible light.

The ability to confine, enhance and control electromagnetic fields in regions of space smaller than or of the order of the excitation wavelength has stirred considerable interest in surface plasmons over the last decade, as it opens the door to a large number of applications in the domains of nanophysics, near-field microscopy, bio-sensing, nanolithography and quantum computing, to name a few.

A great deal of the mathematical work about plasmons has focused on the so-called electrostatic case, where the Maxwell system is reduced to a Helmholtz equation and in the asymptotic limit when the particle diameter is small compared to the frequency ω of the incident wave. After proper rescaling, the study amounts to that of a conduction equation of the form

$$\operatorname{div}\Big(\varepsilon(\omega)^{-1}(x)\nabla u(x)\Big)\Big)=0, \tag{1.1}$$

complemented with appropriate boundary or radiation conditions; see [6,7] for a mathematical justification. The electric permittivity $\varepsilon(\omega)$ in (1.1) takes different forms in the dielectric ambient medium and inside the particle; in the latter situation, it is usually modeled by a Drude-Lorentz law of the form:

$$\varepsilon(\omega)=\varepsilon_0\left(1-\frac{\omega_p^2}{\omega^2+i\omega\gamma}\right),$$

where ε_0 is the electric permittivity of the vacuum and where ω_p and γ respectively denote the plasma frequency and the conductivity of the medium; see [6–8,27,38,39]. In the case of metals such as gold and silver, experimental data show that, for frequencies in the range $200-700$ μm, $\mathrm{Re}(\varepsilon(\omega))<0$, while the rate $\mathrm{Im}(\varepsilon(\omega))$ of dissipation of electrostatic energy is small. In this context, (1.1) gets close to a two-phase conduction equation with sign-changing coefficients and it loses its elliptic character.

In the above electrostatic approximation, the plasmonic resonances of a particle D embedded in a homogeneous medium of permittivity ε_0 are precisely associated with values of the permittivity ε inside the particle for which (1.1) ceases to be well-posed. If the shape of the particle is sufficiently smooth, one may represent the solution u to (1.1) via layer potentials and then characterize plasmon resonances as values of the contrast $\frac{\varepsilon+\varepsilon_0}{2(\varepsilon-\varepsilon_0)}$ which are eigenvalues of the associated Neumann-Poincaré integral operator $\mathcal{K}_D^*$; see [6,38].

Due to their key role in various physical contexts, the spectral properties of the Neumann-Poincaré operator have been the focus of numerous investigations [2,4,13,15,16]. When the inclusion D is smooth (say with $\mathcal{C}^{1,\alpha}$ boundary), $\mathcal{K}_D^*$ is a compact operator and so its spectrum $\sigma(\mathcal{K}_D^*)$ consists in a sequence of eigenvalues that accumulates to 0 [34]. When D is merely Lipschitz, $\mathcal{K}_D^*$ may no longer be compact and $\sigma(\mathcal{K}_D^*)$ may contain essential spectrum–a fact that has motivated several analytical and numerical