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A Characteristics-Mixed Volume Element Method for the Displacement Problem of Enhanced Oil Recovery

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Abstract. The physical model is defined by a coupled system of seepage displacement for simulating chemical oil recovery numerically, formulated by three nonlinear partial differential equations concerning the pressure of Darcy-Forchheimer flow, the concentration and the component saturations. The pressure appears within the concentration equation and saturation equations and the Darcy-Forchheimer velocity controls the concentration and saturations. The flow equation is solved by the conservative mixed finite element method. The order of the accuracy is improved by the velocity. The conservative mixed volume element with characteristics is applied to compute the concentration, i.e., the diffusion is discretized by the mixed volume element and convection is computed by the method of characteristics. The method of characteristics has strong computational stability at sharp fronts and confirms high computational accuracy. The mixed volume element simulates diffusion, concentration and the adjoint vector function simultaneously. The nature of conservation is an important physical feature in the numerical simulation. The saturations of different components are computed by the method of characteristic fractional step difference and the computational work is shortened significantly by decomposing a three-dimensional problem into three successive one-dimensional problems and using the method of speedup. By using the theory and technique of a priori estimates of differential equations, convergence of the optimal second order in l^2 norm is obtained. Numerical examples are provided to show the effectiveness and viability of this method. This composite method can solve the challenging benchmark problem well.

AMS subject classifications: 65N12, 65N30, 65M12, 61M15

Key words: Chemical oil recovery, Darcy-Forchheimer flow, mixed finite element-characteristic mixed volume element, elemental conservation of mass, second-order estimates in l^2-norm.

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1 Introduction

Water-flooding (secondary oil recovery) is effective while much crude oil remains in the reservoirs. The capillary force and other physical factors prevent the oil from migrating and accumulating through the underground media. The behavior of oil-phase is affected by the slight influence of injected fluid in which the mobility ratio is unfavorable. Therefore, chemical addition agents such as polymer, surface active agent and alkali are mixed within the injected fluid to make the oil behavior active. The polymer optimizes the fluidity of displacement phase, modifies the ratio, makes the force at the leading edges well uniform, weakens the process of inner porous layer and increases the influence efficiency and the pressure gradient. Surface active agent and alkali can decrease interfacial tensions of different phases, then make the crude oil migrating and gathering [1–6].

A compositional simulator is addressed in this paper to discuss the chemical oil recovery displacement of Darcy-Forchheimer flow, where a mixed finite element, the characteristics and a mixed volume element are included. Numerical analysis and experimental test are shown, so this discussion gives theoretical and applicable references for numerical simulation of chemical oil recovery. The mathematical model is formulated by the following nonlinear partial differential equations with initial-boundary conditions [1–7]:

$$\mu(c)\kappa^{-1}\mathbf{u}+\beta\rho(c)|\mathbf{u}|\mathbf{u}+\nabla p=r(c)\nabla d, \qquad X=(x,y,z)^T\in\Omega, \quad t\in J=(0,T], \tag{1.1a}$$

$$\nabla\cdot\mathbf{u}=q=q_I+q_p, \qquad X\in\Omega, \quad t\in J, \tag{1.1b}$$

$$\phi\frac{\partial c}{\partial t}+\mathbf{u}\cdot\nabla c-\nabla\cdot(D(\mathbf{u})\nabla c)+q_Ic=q_Ic_I, \qquad X\in\Omega, \quad t\in J, \tag{1.1c}$$

$$\phi\frac{\partial}{\partial t}(cs_\alpha)+\nabla\cdot(s_\alpha\mathbf{u}-\phi c\kappa_\alpha\nabla s_\alpha)=Q_\alpha(X,t,c,s_\alpha), \quad X\in\Omega, \quad t\in J, \quad \alpha=1,\cdots,n_c, \tag{1.1d}$$

where Ω is a bounded domain and $J=(0,T]$ is the time interval. Eq. (1.1) is the Darcy-Forchheimer flow equation, formulated by Darcy-Forchheimer in [8] to describe the motion state of high speed flow in heterogeneous media, especially nearby the wells [9]. If $\beta=0$, then Darcy-Forchheimer's law degrades into Darcy's law. The law of Darcy-Forchheimer was derived in [9] and the regularity was argued in [10].

Eqs. (1.1) and (1.1c) interpret the conservation of mass. The functions $p(X,t)$ and $\mathbf{u}(X,t)$ are the fluid pressure and Darcy-Forchheimer velocity, respectively. $c(X,t)$ denotes the concentration of water phase. $\kappa(X)$, $\phi(X)$ and $\beta(X)$ are the absolute permeability, porosity and Forchheimer coefficient, respectively. $r(X,c)$ is the gravity and $d(X)$ is the vertical coordinate. $q(X,t)$ is the quantity, usually defined by a linear combination of the production q_p and the injection q_I, $q(X,t)=q_I(X,t)+q_p(X,t)$. c_I, a known variable, is the concentration of injected water and $c(X,t)$ is the water concentration at the production well.

In Eq. (1.1d), $s_\alpha=s_\alpha(X,t)$ is the saturation of the αth chemical component, such as the polymer, surfactant, alkali and different ions. n_c is the number of components. For the αth component, $\kappa_\alpha=\kappa_\alpha(X)$ is the diffusion and Q_α is the source and sink term.