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# NUMERICAL SOLUTION OF A TRANSIENT THREE-DIMENSIONAL EDDY CURRENT MODEL WITH MOVING CONDUCTORS

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**Abstract.** The aim of this paper is to propose and analyze a numerical method to solve a time-dependent eddy current problem in a domain containing moving non magnetic conductors. To this end, we choose a formulation in terms of the magnetic field, what leads to a parabolic problem for which we prove an existence result. For space discretization, we propose a finite element method based on Nédélec edge elements on a mesh that remains fixed over the time. The curl-free constraint in the dielectric domain is relaxed by means of a penalty strategy that can be easily implemented, without the need that the mesh fits the moving conducting and dielectric domains. For time discretization, we use a backward Euler scheme. We report some numerical results. First, we solve a test problem with a known analytical solution, which allows us to assess the convergence of the method as the penalization and discretization parameters go to zero. Finally, we solve a problem with cylindrical symmetry, which allows us to compare the results with those obtained with an axisymmetric code.

**Key words.** Eddy current problems, transient electromagnetic problems, moving domains, edge finite elements, penalty formulation.

## 1. Introduction

This paper deals with a finite element method to solve a time-dependent eddy current problem in a three-dimensional (3D) bounded domain containing moving non magnetic conductors. Such a problem arises in different physical applications such as electromagnetic forming process or magnetic levitation. In particular, our work is motivated by the simulation of electromagnetic forming processes (EMF) [8], which leads to solving the transient eddy current model with the conducting part being a workpiece which is deformed over the time due to electromagnetic forces, while the current source arises from a coil placed in a fixed position. A strategy often used in the literature to simulate this process is based on a sequential coupling [11] between an electromagnetic model and a structural one; the former allows computing the Lorentz-forces which drives the motion of the workpiece while the latter uses these forces as data to compute the workpiece deformation. In this way, the mechanical results would allow us to update the geometry to be used in the subsequent step of the electromagnetic model. To perform this coupling it is very useful to have an electromagnetic tool able to consider conducting subdomains whose form and position can change over the time. Thus, in this paper we develop a model for this purpose but we will assume that the geometry and position of the workpiece is known at any time. Our goal is to compute the eddy currents and thereby the Lorentz force in this moving conductor as a first step for a sequential magneto-mechanical coupling.

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The motion of the conductor introduces serious difficulties in the mathematical analysis of the eddy current model, mainly due to the different nature of the equations in the dielectric and conducting parts of the domain. In fact, to the best of our knowledge, there is no result guaranteeing the well-posedness of the 3D continuous problem.

Actually, there are just a few papers dealing with the analysis of the eddy current model considering moving conductors in either two or three dimensions. In particular, a two-dimensional transient eddy current problem arising from modeling electrical engines by considering the rotor motion has been analyzed in [5, 7]. The 3D case has been studied in [6], where a time-primitive of the electric field has been used as the main unknown leading to a degenerate parabolic problem. We notice that in these papers the interface between the moving and the fixed part is always the same. Moreover, the fact that the motion is a rotation is used in the theoretical proofs of existence and uniqueness of solution. Such a special kind of motion is quite different to what happens in other processes like EMF or magnetic levitation. From the point of view of the numerical solution, the techniques used in these papers are based on using different reference frames in the moving and non-moving parts, which involve Lagrangian formulations: this leads to work with independent meshes at each part of the domain, while the coupling transmission conditions are taken into account by using mortar techniques.

On the other hand, an axisymmetric eddy current model with workpiece motion has been more recently studied in [3, 4]. The main unknown in this case is a magnetic vector potential and the resulting problem is also parabolic and degenerate; the well-posedness of the problem is proved by means of a regularization argument. In this case, the problem is studied by using a unique reference frame. From the numerical point of view the proposal in these papers is to work with a fixed mesh over the whole time interval, even though the workpiece changes its position. This procedure is based on using low-order quadrature rules with a large number of integration points in those terms involving piecewise smooth discontinuous functions which appear due to the motion of the workpiece.

In this paper, we are interested in 3D problems where the conducting piece is not magnetic and moves freely in the dielectric domain and its motion is not necessarily rigid. From the mathematical point of view, to apply the above discussed results to this kind of problems does not seem to be feasible. In fact, the techniques from [6] do not seem to be applicable to these problems, because the geometry changes arbitrarily over the time. On the other hand, the approach from [3] is based on the fact that the cylindrical symmetry leads to a two-dimensional problem for a scalar variable and the proofs rely on a specific Reynold's transport theorem which does not hold in the present case.

Among the variety of possible formulations (see, for instance, [2]), for our choice we have prioritized three aspects: (i) the possibility of using a fixed mesh of the whole domain at all time, (ii) to avoid the need of building cutting surfaces (what can be extremely cumbersome in complex topologies) and (iii) to use a number of unknowns as small as possible. According to this, we have chosen a formulation in terms of the magnetic field which only involves this vector unknown in the whole domain. Let us remark that an alternative formulation with similar features could be based on a primitive of the electric field without using a gauge condition in the dielectric domain (see again [6]). The ideas used in the present paper regarding how to deal with a fixed mesh could be also tried in this case, although this choice would lead to solving a system with a singular matrix. Let us remark that we will not