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Computation of Shape Derivatives in Electromagnetic Shaping by Algorithmic Differentiation

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Abstract. Shape optimization based on analytical shape derivatives is meanwhile a well-established tool in engineering applications. For an appropriate discretization of the underlying problem, the technique of algorithmic differentiation can also be used to provide a discrete analogue of the analytic shape derivative. The present article is concerned with the comparison of both types of gradient calculation and their effects on a gradient-based optimization method with respect to accuracy and performance, since so far only a few attempts have been made to compare these approaches. For this purpose, the article discusses both techniques and analyses the obtained numerical results for a generic test case from electromagnetic shaping. Since good agreement of both methods is found, algorithmic differentiation seems to be worthwhile to be used also for more demanding shape optimization problems.

AMS subject classifications: 49M25, 49Q10, 78M15

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1 Introduction and motivation

Gradient-based optimization methods are frequently used in engineering applications. In particular, shape optimization is quite indispensable for designing and constructing in-

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dustrial components. Many problems that arise in application, particularly in structural mechanics and in the optimal control of distributed parameter systems, can be formulated as the minimization of functionals defined over a class of admissible domains.

Analytic shape optimization methods are known to be an efficient numerical tool for free boundary computations in electromagnetic shaping, see e.g. [4, 6, 11, 29, 34–36]. In [15, 16], the first two authors developed first and second order algorithms for elliptic shape optimization problems with additional functional constraints. A wavelet-based boundary element method was used for the computation of the objective and related first and second order shape derivatives. These algorithms have successfully been applied in [17] for exterior electromagnetic shaping.

As an alternative approach to provide derivatives one may use Algorithmic Differentiation (AD). AD yields exact derivative information for a function evaluation given as computer code. A comprehensive introduction to AD can be found in [23].

In the context of optimal control problems, the forward mode of AD can be seen as a discrete version of the sensitivity approach. Conversely, the reverse mode of AD involves a discrete adjoint somehow related to the continuous adjoint equation. Despite the fact that these parallels have already been hinted at in [22], a detailed theoretical analysis of the relations between the exact discrete derivatives provided by AD and the corresponding continuous derivative formulation is only available for optimal control problems based on ODEs, see e.g. [19, 41]. The influences of the different derivative information, i.e., either the exact discrete derivatives of the evaluation program, provided by AD, or the continuous derivatives provided by the adjoint equations, on the whole optimization problem were studied in [21] for small ODE-based optimization problems. In [31], the gradient computation for a rather small two-dimensional optimization problem based on Navier-Stokes equations was considered using a Taylor-Hood finite element discretization in space and an implicit Euler scheme in time.

The aim of the present article is to use first the analytic gradient derived from the analytical setting. For the implementation of both, the objective and the analytical gradient, a wavelet-based boundary element method is chosen. Subsequently, it will be shown that also the application of AD is feasible for this problem of medium to large-scale size. Finally, the whole optimization process obtained with the analytic gradient is compared with similar computations based on gradients generated by the AD tool ADOL-C [42].

This article is organized as follows. Section 2 repeats the main aspects about the underlying model and analytic gradients in electromagnetic shaping. The discretization of the unknown shape, the numerical solution of the boundary integral equation by a wavelet-based boundary element method, and the optimization method for the resulting finite dimensional optimization problem are discussed in Section 3. In Section 4, we recall basic facts about the concept of algorithmic differentiation and its implementation. Finally, Section 5 summarizes our comparison tests with various respects. In Section 6, we state concluding remarks.