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Further Study on Errors in Metric Evaluation by Linear Upwind Schemes with Flux Splitting in Stationary Grids

Qin Li[1,2,*], Dong Sun[1] and Pengxin Liu[1]

[1] *State Key Laboratory of Aerodynamics, Mianyang, Sichuan, 621000, P.R. China.*
[2] *National Laboratory of Computational Fluid Dynamics, Beijing University of Aeronautics and Astronautics, Beijing 100191, P.R. China.*

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Abstract. The importance of eliminating errors in grid-metric evaluation for high-order difference schemes has been widely recognized in recent years, and it is known from the proof by Vinokur and Yee (NASA TM 209598, 2000) that when conservative derivations of grid metric are used by Thomas, Lombard and Neier (AIAA J., 1978, 17(10) and J. Spacecraft and rocket, 1990, 27(2)), errors caused by metric evaluation could be eliminated by linear schemes when flux splitting is not considered. According to the above achievement, central schemes without the use of flux splitting could fulfill the requirement of error elimination. Difficulties will arise for upwind schemes to attain the objective when the splitting is considered. In this study, further investigations are made on three aspects: Firstly, an idea of central scheme decomposition is introduced, and the procedure to derive the central scheme is proposed to evaluate grid metrics only. Secondly, the analysis has been made on the requirement of flux splitting to acquire free-stream preservation, and a Lax-Friedrichs-type splitting scheme is proposed as an example. Discussions about current study with that by Nonomura et al. (Computers and Fluids, 2015, 107) have been made. Thirdly, for half-node- or mixed-type schemes, interpolations should be used to derive variables at half nodes. The requirement to achieve metric identity on this situation is analyzed and an idea of directionally consistent interpolation is proposed, which is manifested to be indispensable to avoid violations of metric identity and to eliminate metric-caused errors thereafter. Two numerical problems are tested, i.e., the free-stream and vortex preservation on wavy, largely randomized and triangular-like grids. Numerical results validate aforementioned theoretical outcomes.

AMS subject classifications: 65M06

Key words: Upwind scheme, flux splitting, metric identity, free-stream preservation.

*Corresponding author. *Email addresses:* qin-li@vip.tom.com (Q. Li), sund1986@foxmail.com (D. Sun), liupengxin2008@163.com (P. Liu)

1 Introduction

It is well-known in computational fluid dynamics (CFD) that the use of deformed grids usually leads to unsatisfactory results. In Ref. [1], Visbal and Gaitonde demonstrated considerable errors might be caused by metric evaluations when using high-order schemes. Later, the numerical investigations by Nonomura, Lizuka and Fujii [2] again verified the importance of the metric computation. Through their work, the issue regarding metric-caused errors has re-gained the attention of CFD community.

At least in 1974, Vinokur [3] gave the conservative forms of Euler Equations in stationary curvilinear coordinate systems, which implied the use of theoretically zero-valued terms, i.e., metric identities. In 1978, Pulliam and Steger [4] pointed out that the presumed zero-valued identities might actually have non-zero value in computations. Hence when the uniform flow condition is imposed, the flow field might change and so-called free-stream preservation (*FSP*) property could be broken. In the following, a brief discussion is made on the efforts to eliminate errors generated in metric evaluations in three aspects:

(1) The form of grid metric. In CFD textbooks, grid metrics are usually expressed in products of coordinate derivatives, e.g., $\hat{\xi}_x = y_\eta z_\zeta - z_\eta y_\zeta$. When this form is chosen, it seems that only second-order schemes with averaging technique [4] can achieve metric cancellation and make metric identity (*MI*) established. Using simple re-combination, Thomas and Lombard [5, 6] proposed a "conservative" form of the metrics, through which the restriction of using specific difference scheme to achieve metric cancellation was largely released. Thomas and Neier [7] further recast the conservative form into a more symmetric one, which later was referred by Vinokur and Yee [8] as the "coordinate invariant form".

(2) The practice of using the same scheme for the metric and flux derivatives in fluid governing equations. In Ref. [9], Thompson et al. mentioned that it would be better to use the same difference representation to evaluate the metric coefficients and the function such as flux derivatives. Gaitonde and Visbal [10] explicitly stated that metrics "computed with the same scheme as employed for the fluxes" could reduce "the error on stretched meshes".

(3) The approaches to avoid errors lead by grid metrics. After numerically testing various center schemes with orders from the second to sixth, Gaitonde and Visbal [10] found the coupling of the same-scheme practice with the conservative form of metrics in Ref. [6] could reduce metric-caused errors to machine zero. Vinokur and Yee [8] realized the key lay in the numerical commutativity of the mixed partial derivative and showed an analytic proof on the commutativity by using the notion of tensor product. Later, different analyses were conducted on the same subject from different aspects [11–14]. Besides the above methods, other efforts were observed such as positively removing the errors introduced during the equation transformations. This idea could be found in Ref. [9] and [15], and Cai and Ladeinde [16] showed a numerical practice of this regard. It is usually suspected whether such practice would be a thorough solution to the problem.