COMMUNICATIONS IN MATHEMATICAL RESEARCH
33(3)(2017), 223–237

On C-left Hyperideals of Ordered Semihypergroups

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Communicated by Du Xian-kun

Abstract: In this paper, the concept of C-left hyperideals is introduced in ordered semihypergroups, and several related properties are investigated. In particular, we discuss the relationships between the greatest left hyperideals and C-left hyperideals of ordered semihypergroups. Furthermore, we introduce the concept of left bases of an ordered semihypergroup, and give out the sufficient and necessary conditions of the existence of the greatest C-left hyperideal of an ordered semihypergroup by the properties of left bases. The result about C-left ideals in ordered semigroups is generalized to the ordered semihypergroups.

Key words: ordered semihypergroup, C-left hyperideal, maximal left hyperideal, left base

2010 MR subject classification: 20N20, 06F05

Document code: A

Article ID: 1674-5647(2017)03-0223-15

DOI: 10.13447/j.1674-5647.2017.03.04

1 Introduction

In mathematics, an ordered semigroup is a semigroup together with a partial order that is compatible with the semigroup operation. Ordered semigroups have several applications in the theory of sequential machines, formal languages, computer arithmetics and error-correcting codes. There are several results which have been added to the theory of ordered semigroups by several researchers. For more details, the reader is referred to [1]–[6]. In

Received date: Feb. 25, 2016.

Foundation item: The NSF (11271040, 11361027) of China, the Key Project (2014KZDXM055) of Department of Education of Guangdong Province, the University Natural Science Project (KJ2015A161) of Anhui Province and the NSF (2014A030313625) of Guangdong Province.

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particular, Kehayopulu[1],[3] introduced the concepts of ideals, prime ideals and weakly prime ideals of ordered semigroups, and provided some results which are similar to that in ring theory. In [4] and [6], Xie *et al.* defined and studied the C-ideals and C-left ideals in ordered semigroups.

On the other hand, algebraic hyperstructures, particularly hypergroups, were introduced by Marty[7] in 1934. Later on, algebraic hyperstructures have been intensively studied, both from the theoretical points of view and especially for their applications in other fields (see [8]). One of the main reasons which attracts researchers towards algebraic hyperstructures is its unique property that in algebraic hyperstructures composition of two elements is a set, while in classical algebraic structures the composition of two elements is an element. Thus algebraic hyperstructures are a suitable generalization of classical algebraic structures. The study on the theory of semihypergroups is one of the most active subjects in algebraic hyperstructure theory. A theory of hyperstructures on ordered semigroups has been recently developed. It is worth pointing out that Heidari and Davvaz[9] applied the theory of hyperstructures to ordered semigroups and introduced the concept of ordered semihypergroups, which is a generalization of the concept of ordered semigroups. Later on, Changphas and Davvaz[10] investigated the properties of hyperideals in ordered semihypergroups in detail. In [11], Tang *et al.* introduced the concept of hyperfilters of ordered semihypergroups, and discussed the relationships between hyperfilters and completely prime hyperideals of ordered semihypergroups. As a further study of hyperideals in ordered semihypergroups, in this paper we attempt to introduce and give a detailed investigation of C-left hyperideals of an ordered semihypergroup. Especially, we discuss the relationship between the greatest left hyperideals and C-left hyperideals of ordered semihypergroups. Furthermore, we introduce the concept of left bases of an ordered semihypergroup, and provide the sufficient and necessary conditions of the existence of the greatest C-left hyperideal of an ordered semihypergroup in terms of left bases. As an application of the results of this paper, the corresponding results of semihypergroups (without order) and ordered semigroups are also obtained by moderate modifications.

2 Preliminaries and Some Notations

Recall that a hypergroupoid $(S,\ \circ)$ is a nonempty set S together with a hyperoperation, that is a map $\circ : S \times S \to P^*(S)$, where $P^*(S)$ denotes the set of all the nonempty subsets of S. The image of the pair $(x,\ y)$ is denoted by $x \circ y$. If $x \in S$ and $A,\ B$ are nonempty subsets of S, then $A \circ B$ is defined by

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b.$$

Also $A \circ x$ is used for $A \circ \{x\}$ and $x \circ A$ for $\{x\} \circ A$. Generally, the singleton $\{x\}$ is identified by its element x.

We say that a hypergroupoid $(S,\ \circ)$ is a semihypergroup if the hyperoperation " $\circ$ " is