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On Fundamental Group of a Certain Class of Welded Knots

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Abstract: In this paper, a certain class of welded knots K_{2n} is considered. By calculating the commutators subgroup of fundamental group G_n of welded knot K_{2n}, $n \in \mathbf{Z}^+$, we show that these welded knots are not equivalent to each other and they are all not classical knots. Secondly, we study some properties of G_n and obtain that G_n is linear, residually finite and Hopfian.

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1 Introduction

In 1996, Kauffman[1] introduced the notion of a virtual knot, which is motivated by the study of knots in a thickened surface and abstract Gauss codes. Goussarov *et al.*[2] showed that virtual knot theory is very effective and natural to study in relation to finite type invariants of knots. According to [2], two classical knot diagrams represent the same knot type if and only if they represent the same virtual knot type, which is also proved by Kauffman[1] by using the notion of involution quandle. Hence the concept of virtual knot is a generalization to classical knot in three space.

Several invariants in classical knot theory can be naturally extended to virtual knot, including the Jones polynomial and knot group $G(K)$ (see [1]). However, the Jones polynomial is not well-defined for welded knot. Fish and Keyman[3] defined a Jones polynomial invariant for fused links by changing the base ring $\mathbb{Z}[t^{\pm 1}]$ to some quotient ring. But for welded knot, this Jones polynomial invariant is still trivial. As far as we are concerned, little

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is known about polynomial invariant for welded knot theory. Hence, for welded knot theory, the fundamental group of a welded knot is the main invariant so far.

In 2000, Silver and Williams[4] gave an example of virtual knot which is not a classical knot by discussing the commutators subgroup of its fundamental group.

In this paper, we consider a certain class of welded knot whose diagram was shown in Fig. 1.1, which is a generalization of the example of Silver and Williams, and denote this welded knot by K_{2n}.

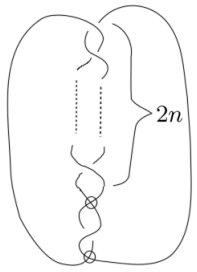

Fig. 1.1 The welded knot K_{2n}

This welded knot can be regarded as by flanking one classical crossing from torus knot $T(2n+1,2)$ by virtuals and switching its crossing. For convenience, We denote the fundamental group of K_{2n} by G_n. By calculating the fundamental groups of these welded knots for different n, we prove that they are pairwise different and they are all not classical knots. Main results are the following:

Lemma 1.1 $G_n = \langle a, b \mid \underbrace{bababab\cdots ab}_{2n+1} = \underbrace{abababa\cdots ba}_{2n+1},\ a^2 = b^2 \rangle,\ n \in \mathbf{Z}^+$.

Lemma 1.2 $[G_n, G_n] \cong \mathbb{Z}_{2n+1},\ n \in \mathbf{Z}^+$.

Theorem 1.1 *$K_{2m} \neq K_{2n}$ as welded knot if $m \neq n$ $(m, n \in \mathbf{Z}^+)$.*

Corollary 1.1 *K_{2n} is not a classical knot for $n \in \mathbf{Z}^+$.*

This paper is organized as follows. In Section 2, we recall some definitions about virtual and welded knot theory. In Section 3, we give the presentation of the fundamental group of welded knot K_{2n}. By calculating the commutators subgroup of the fundamental group G_n of K_n, we show that K_n is different to K_m if m is different to n. Finally, for G_n is something like between Artin group and Coxeter group, we study some properties of G_n and obtain that G_n is a linear group, residually finite and hopfian.

2 Virtual Knot and Welded Knot Theory

A shape virtual knot diagram is a generic immersion of the circle into the plane, with double points representing classical crossing and virtual crossings. The branches of a classical