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Stable Semi-Implicit Monolithic Scheme for Interaction Between Incompressible Neo-hookean Structure and Navier-Stokes Fluid

Cornel Marius Murea*

Département de Mathématiques, Institut de Recherche en Informatique, Université de Haute Alsace, 6, rue des Frères Lumière, 68093 Mulhouse Cedex, France.

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Abstract. We present a monolithic algorithm for solving fluid-structure interaction. The Updated Lagrangian framework is used for the incompressible neo-hookean structure and Arbitrary Lagrangian Eulerian coordinate is employed for the Navier-Stokes equations. The algorithm uses a global mesh for the fluid-structure domain which is compatible with the fluid-structure interface. At each time step, a non-linear system is solved in a domain corresponding to the precedent time step. It is a semi-implicit algorithm in the sense that the velocity, the pressure are computed implicitly, but the domain is updated explicitly. Using one velocity field defined over the fluid-structure mesh, and globally continuous finite elements, the continuity of the velocity at the interface is automatically verified. The equation of the continuity of the stress at the interface does not appear in this formulation due to action and reaction principle. The stability in time is proved. A second algorithm is introduced where at each time step, only a linear system is solved in order to find the velocity and the pressure. Numerical experiments are presented.

AMS subject classifications: 74F10, 65M12

Key words: Fluid-structure interaction, monolithic algorithm, stability.

1 Introduction

We can solve numerically fluid-structure interaction problems by partitioned procedures or monolithic algorithms and a large literature exists in this subject. In some monolithic formulations [12, 14], two non-overlapping meshes are used for fluid and structure domains and the boundary conditions at the fluid-structure interface appear as equations in the global system.

*Corresponding author. *Email address:* `cornel.murea@uha.fr` (C.M. Murea)

Other monolithic formulations use Eulerian mesh which does not fit to the fluid-structure interface. In [6, 7, 22], an Eulerian approach is used for the fluid as well as for the structure and the interface is captured with Initial Point Set. Extended Finite Element Method (XFEM) was used in [9]. In [2, 3], fictitious domain method with Lagrange multiplier was employed where the structure is assumed to be visco-elastic. This assumption is used in [26], too. Explicit schemes for fluid-structure interaction problems using Nitsche's method and RobinRobin coupling are discussed in [4]. The stability is proved under a hyperbolic type CFL condition.

In [11, 16, 18-20, 24] one global mesh for fluid-structure domain which fits to the interface is used. In [11, 20], an Eulerian formulation derived from Cayley-Hamilton theorem is used for the incompressible Mooney-Rivlin structure. The fluid equations are solved by the Characteristics-Galerkin method. The fluid-structure equations are written in the unknown Eulerian domain and fixed-point iterations are performed at each time step. The authors prove the time stability of the scheme.

In [16] where the structure is linear elastic and in [18] where the structure is compressible neo-hookean, the Updated Lagrangian coordinates are used for the structure combined with the Arbitrary Lagrangian Eulerian framework for the fluid equations. Using one velocity field defined over the fluid-structure mesh, and globally continuous finite elements, the continuity of the velocity at the interface is automatically verified. The equation of the continuity of the stress at the interface does not appear in this formulation due to action and reaction principle. Another advantage of this approach is that the fluid-structure equations are written in the known domain obtained at the precedent time step. It is a semi-implicit algorithm in the sense that the fluid and structure unknowns are computed implicitly, but the time advancing scheme for the domain is explicitly.

We follow this idea in the present paper, but for incompressible neo-hookean structure with a different computation of the mesh velocity. The structure equations in Updated Lagrangian coordinates is well posed. The stability in time of the monolithic algorithm is proved. The system to be solved at each time step can be easely linearised. Numerical results are presented in the last section.

2 Fluid-structure interaction problem

Without restriction of generality, we consider the geometrical configuration of the benchmark FSI3 from [25]. The results presented in this paper, including the stability analysis, hold for different 2D geometrical configurations, for example the flow in a channel with elastic wall, [16]. For 3D configurations, for example blood flow in artery [19], the stability result for the structure remains true but the stabilization term added in the fluid scheme has to be adapted accordingly.

We consider a rectangular flexible structure of length ℓ and thickness h immersed in an incompressible fluid occupying the rectangular domain $(0,L)\times(0,H)$. The rectangular structure is attached to a fixed body of boundaries: the segment $[DE]$, which is the left