Communications in Mathematical Research
34(3)(2018), 230–240

A Note on Homogenization of Parabolic Equation in Perforated Domains

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Communicated by Wang Chun-peng

Abstract: We are concerned with a class of parabolic equations in periodically perforated domains with a homogeneous Neumann condition on the boundary of holes. By using the periodic unfolding method in perforated domains, we obtain the homogenization results under the conditions slightly weaker than those in the corresponding case considered by Nandakumaran and Rajesh (Nandakumaran A K, Rajesh M. Homogenization of a parabolic equation in perforated domain with Neumann boundary condition. Proc. Indian Acad. Sci. (Math. Sci.), 2002, 112(1): 195–207). Moreover, these results generalize those obtained by Donato and Nabil (Donato P, Nabil A. Homogenization and correctors for the heat equation in perforated domains. Ricerche di Matematica L. 2001, 50: 115–144).

Key words: parabolic equation, perforated domain, homogenization, periodic unfolding method

2010 MR subject classification: 35B27, 35K55

Document code: A

Article ID: 1674-5647(2018)03-0230-11

DOI: 10.13447/j.1674-5647.2018.03.05

1 Introduction

In this paper, we study the homogenization for the following problem:

$$\begin{cases} \rho_\varepsilon u_\varepsilon' - \operatorname{div}(A^\varepsilon \nabla u_\varepsilon) = f & \text{in} \quad \Omega_\varepsilon^* \times (0, T), \\ u_\varepsilon = 0 & \text{on} \quad \partial\Omega \times (0, T), \\ A^\varepsilon \nabla u_\varepsilon \cdot n_\varepsilon = 0 & \text{on} \quad \partial S_\varepsilon \times (0, T), \\ u_\varepsilon(x, 0) = u_0 & \text{in} \quad \Omega_\varepsilon^*, \end{cases} \tag{1.1}$$

Received date: June 14, 2017.
Foundation item: The NSF (11401595) of China and the Nationalities Innovation Foundation (2018sycxjj113) of South-central University for Postgraduate.
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where $\Omega \subset \mathbf{R}^n$ is an open and bounded set, $\Omega_\varepsilon^* = \Omega \backslash S_\varepsilon$ is a domain perforated by S_ε which is a set of ε-periodic holes of size ε, and n_ε is the outward unit normal vector field defined on ∂S_ε. The initial data u_0 and f belong to $L^\infty(\Omega)$ and $L^2(0, T; L^2(\Omega))$, respectively. The matrix $A^\varepsilon(x)$ is of form $A\left(\frac{x}{\varepsilon}\right)$ with A being a periodic, bounded and elliptic matrix field. We also assume $\rho_\varepsilon = \rho\left(\frac{x}{\varepsilon}\right)$ with ρ being a Y-periodic and coercive function in $L^\infty(\Omega)$. This problem is widely used to model many phenomena in the heat theory.

The study of this problem was initiated by the work of Spagnolo[1], in which he achieved the homogenization of problem (1.1) for fixed domains. Further investigations were made by Bensollssa *et al.*[2]. The corresponding correctors were achieved in [3]. Subsequently, much attention has been paid to such an investigation for some different cases, see [4]–[16] and the references therein. Recently, Meshkova and Suslina[17] studied the homogenization of the second initial boundary value problem for parabolic systems with rapidly oscillating coefficients. Chakib *et al.*[18] investigated the periodic homogenization of nonlinear parabolic problem, which is defined in periodical domain and is nonlinear at the interface. Amar *et al.*[19] obtained the homogenization of a parabolic problem in a perforated domain with Robin-Neumann boundary conditions oscillating in time. In particular, for the case in periodically perforated domains, Donato and Nabil carried out a study of the homogenization and correctors for the standard linear case in [20] and for the semi-linear case in [6], respectively. Then, Donato and Yang[7] extended the results in [20] to the case with non-periodic coefficients. In [13], Nandakumaran and Rajesh studied the following nonlinear degenerate problem

$$\begin{cases} \partial_t b\left(\frac{x}{\varepsilon}, u_\varepsilon\right) u_\varepsilon' - \operatorname{div} a\left(\frac{x}{\varepsilon}, u_\varepsilon, \nabla u_\varepsilon\right) = f & \text{in } \Omega_\varepsilon^* \times (0,T), \\ u_\varepsilon = 0 & \text{on } \partial\Omega \times (0,T), \\ a\left(\frac{x}{\varepsilon}, u_\varepsilon, \nabla u_\varepsilon\right) \cdot n_\varepsilon = 0 & \text{on } \partial S_\varepsilon \times (0, T), \\ u_\varepsilon(x, 0) = u_0 & \text{in } \Omega_\varepsilon^*. \end{cases} \tag{1.2}$$

To obtain the homogenization results, they imposed some extra condition on $\widetilde{u}_\varepsilon$ ($\widetilde{u}_\varepsilon$ is uniformly bounded in $L^\infty(0, T; L^2(\Omega))$). The proof mainly depends on the two-scale convergence method. For the corresponding case in fixed domains, the homogenization and the correctors were provided in [12] and given for the case $b\left(\frac{x}{\varepsilon}, u_\varepsilon\right) \equiv b(u_\varepsilon)$ in [11].

In this paper, we are devoted to obtaining the homogenization of the problem (1.1) under the conditions slightly weaker than those in the corresponding case of [13]. Moreover, our results generalize the work in [20]. Our method mainly relies on the periodic unfolding method, which was originally introduced in [21] (see also [22]) and extended to perforated domains in [23] (see [24] for more general situations). Recently, the unfolding technique was extended to the time-periodic case. Then it was further used to consider the homogenization problem for a parabolic equation oscillating both in space and time, with general independent scales.

Throughout this paper, we make the following assumptions: