

## Linear Wavefield Optimization Using a Modified Source

Tariq Alkhalifah<sup>1,\*</sup>

<sup>1</sup> *Physical Sciences and Engineering, King Abdullah University of Science and Technology, Mail box # 1280, Thuwal 23955-6900, Saudi Arabia.*

Received 28 May 2018; Accepted (in revised version) 9 October 2019

---

**Abstract.** Recorded seismic data are sensitive to the Earth's elastic properties, and thus, they carry information of such properties in their waveforms. The sensitivity of such waveforms to the properties is nonlinear causing all kinds of difficulties to the inversion of such properties. Inverting directly for the components forming the wave equation, which includes the wave equation operator (or its perturbation), and the wavefield, as independent parameters enhances the convexity of the inverse problem. The optimization in this case is provided by an objective function that maximizes the data fitting and the wave equation fidelity, simultaneously. To enhance the practicality and efficiency of the optimization, I recast the velocity perturbations as secondary sources in a modified source function, and invert for the wavefield and the modified source function, as independent parameters. The optimization in this case corresponds to a linear problem. The inverted functions can be used directly to extract the velocity perturbation. Unlike gradient methods, this optimization problem is free of the Born approximation limitations in the update, including single scattering and cross talk that may arise for example in the case of multi sources. These specific features are shown for a simple synthetic example, as well as the Marmousi model.

**AMS subject classifications:** 86A15, 65K10

**Key words:** Inversion, optimization, scattering theory.

---

## 1 Introduction

Recording waves that may originate from active, or natural sources, including ambient noise is now prevalent in many applications ranging from medical imaging, reverse engineering, non-destructive testing, and, of course, delineating the Earth physical properties. The resulting signals carry information of the object they originated from and the medium they travelled through. The state of these waves as a function of space and time are referred to as wavefields. These functions depend on the source of the wavefield energy and the medium they reside in [2]. A special kind of wavefield is the Green's

---

\*Corresponding author. *Email address:* tariq.alkhalifah@kaust.edu.sa (T. Alkhalifah)

function [7], which represents the wavefield response to a specific point source in time and space (or just space in most practical applications, considering our band limited signals). So, wavefields tend to be a superposition or summation of these Green's functions weighted by the actual sources of energy in the wavefield, as well as the sources of scattering (secondary sources) [8,9,20]. In real life, wavefields are only known at the sensor (recording device) locations. In our computing devices, we solve for these wavefields using the appropriate wave equation (considering the physical nature of the medium), for a given source of energy (location and structure) and given medium properties. If within the simulation process of waves, the source or the medium properties are not representative of the true source or medium properties under investigation, the wavefield would usually be wrong and its values at the simulated sensors would differ from those measured in the real experiment. In classic waveform inversion, we use such differences, measured in many ways, to update the source information and the medium parameters or at least one of them [15]. An integral part of this process is the accuracy of the wavefield, which connects these unknowns to the measurements, and often satisfies a particular wave equation, or specifically its partial differential equation (PDE) form in time or frequency. For the specific problem of waves propagating within a medium, having the accurate wave equation for a specific medium, implies having the accurate form, the medium information, the wavefield and source function. The classic inversion method suffers from the sinusoidal nature of waves, and thus, faces issues related to cycle skipping and the highly nonlinear relation between the medium properties and the wave behavior. Improvements in the performance of waveform inversion is crucial to many applications as the cost of the process is high [5,14,18,19].

An approach to reduce the nonlinearity of waveform inversion is provided by loosening the constraint on the wave equation and allowing the wavefield to fit the data regardless of the velocity model [1,11,17,23]. As a result, the optimization problem includes, at least, two terms, or two objectives: reducing the modelled wavefield misfit to the data and increasing its compliance to the wave equation. Using such an optimization, [16] and [1] invert for the medium perturbations and the source contrasts. On the other hand, [17] elected to invert for the wavefield and the medium perturbations. The philosophy behind both approaches is supported by the inversion iterative nature. Since the initial velocity model is assumed wrong (it provides wavefields that do not fit the data), then why do we need to constrain the wavefield to the wave equation. The wave equation is as good as its operator, which is driven primarily by the model. However, in both implementations, updating the velocity model is an integral part of the iterative process.

Since the scattering series, and specifically, the Lippmann-Schwinger equation [21] suggests that the wavefield can be constructed from the background model and scattering (secondary or contrast sources), we can formulate an optimization for the wavefield and the secondary sources, and initially bypass inverting for the source of nonlinearity given by the medium perturbations. The inversion for the medium perturbations can happen in a follow up step. Thus, in this case, the wave equation operator remains stationary,