Commun. Math. Res.
doi: 10.4208/cmr.2020-0011

Vol. **36**, No. 2, pp. 211-228
May 2020

An Ensemble Kalman Filter Approach Based on Level Set Parameterization for Acoustic Source Identification Using Multiple Frequency Information

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Received 28 January 2020; Accepted 1 May 2020

Abstract. In this paper, a reconstruction problem of the spatial dependent acoustic source from multiple frequency data is discussed. Suppose that the source function is supported on a bounded domain and the piecewise constant intensities of the source are known on the support. We characterize unknown domain by the level set technique. And the level set function can be modeled by a Hamilton-Jacobi system. We use the ensemble Kalman filter approach to analyze the system state. This method can avoid to deal with the nonlinearity directly and reduce the computation complexity. In addition, the algorithm can achieve the stable state quickly with the Hamilton-Jacobi system. From some numerical examples, we show these advantages and verify the feasibility and effectiveness.

AMS subject classifications: 35R20, 65R20

Key words: Level set, data assimilation, acoustic source, EnKF.

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1 Introduction

The unknown source problems occur widely in many real applications, e.g., remote sensing, radar detection [16], brain image etc. In this paper, we consider

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a reconstruction problem of acoustic sources from remote measurements of the acoustic field. For a spatial dependent acoustic source $f(x)$ supported on region D, the pressure u of the radiated time-harmonic wave can be modeled [9] by

$$(\triangle+k^2)u(x,k)=f(x), \quad x\in\mathbb{R}^d, \tag{1.1}$$

$$\lim_{r\to\infty} r^{\frac{d-1}{2}}\left\{\frac{\partial u}{\partial r}-iku\right\}=0, \quad r=|x|, \tag{1.2}$$

where $k=\omega/c_0$ is the wave number, ω is the radial frequency, c_0 is the speed of sound, $d=2$ is the spatial dimension and (1.2) is the Sommerfeld radiation condition. It is emphasized that D may consist of (finitely many) multiple components. The source f is usually unknown and needs to be reconstructed from some observed data. The data receiver devices are generally located on a remote closed surface $\partial\Omega$, and the interior domain Ω is supposed to contain the support set D. In this paper, we assume that f is of the form

$$f(x)=\sum_{l=1}^{n} w_l \mathbb{I}_{D_l}(x); \tag{1.3}$$

here $\mathbb{I}_D$ denotes the indicator function of subset $D\subset\mathbb{R}^2$, $\{D_l\}_{l=1}^n$ are subsets of D such that $\bigcup\limits_{l=1}^{n}\bar{D}_l=\bar{D}$ and $D_l\cap D_j=\varnothing$ $(l\neq j)$, the $\{w_l\}_{l=1}^n$ are known positive constants. Then regions D_l need to be determined to give an reconstruction of f.

This reconstruction problem has been discussed in [2, 9] and the source f is determined uniquely according to the multiple frequency data. Many very effective approaches have been proposed to solve the reconstruction problem of the acoustic source [3, 10, 25]. The main techniques in these works include iteration and sampling algorithms. And the ill-posedness is usually addressed through the regularization technique.

For geometry shape problems, a very powerful tool is the level set method [18], which was originally introduced by Osher and Sethian [19]. It is devised as a simple and versatile method for computing and analyzing the motion of an interface. The approach allows for topological changes to be detected during the course of algorithms, which is impossible with classical methods based on curve parameterizations. Algorithms based on it have also been widely applied in some inverse problems involving shape reconstruction [1,4,7,15,23,24].

In this paper, we propose an ensemble Kalman filter (EnKF) algorithm to find the approximated level set function, which is established on the Bayesian inversion [22]. To the authors' knowledge, there are few studies about this. In [5, 13],