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Mathematical Analysis of an Obesity Model with Eating Behaviors

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Abstract. Overweight is a social disease, which is transmitted through social networks. A mathematical model is proposed to simulate the dynamics of social obesity, where the structures of individual heterogeneity and overeating behaviors are incorporated. The basic reproduction number of the disease is calculated and is shown to be a threshold for disease invasion. Sufficient conditions for the global stability of an endemic equilibrium is established by Lyapunov functions. Numerical simulations are provided to reveal how interventions through treatment to eating behaviors and eduction to susceptible individuals suppress the progression of the disease.

AMS subject classifications: 92D30, 34D23

Key words: Overeating behavior, reproduction number, global stability, intervention.

1 Introduction

Obesity is a social disease that influences not only the health of human individuals, but also imposes the social burdens [20, 22, 25, 27, 28]. It is believed that eating behaviors through social networks and genetic factors contribute to overweight. First, relatives and friends facilitate the transmission of overeating patterns through social dinners and so on. Indeed, people tend to eat more when their eating companions eat more. Second, it is estimated that genetic factors account for 40-90% of the population variation in body mass index (BMI) [9]. There have been some mathematical researches in the aim to reveal mechanisms of propagation of obesity [1,6,8,11,17,18,21,22]. Basically, these models divide the population into the compartments of normal population, overweight population and obesity individuals and consider the contagious transmission between normal individuals and overweight individuals. However, genetic contributions to overweight are neglected in these models. Note that both eating habits and inherent factors determine

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the obesity of an individual. In this paper, we improve the modeling of overweight and obesity by incorporating the heterogeneity of population due to genetic factors, where the population is split into two subpopulations: each member in the first one has innately a normal weight, and every individual in the second class has innately an overweight body.

The organization of this paper is as follows. In the next section, we present the model formulations. Section 3 gives the qualitative analysis. In Section 4, we provide numerical simulations to demonstrate the effects of intervention measures. The paper ends with short discussions.

2 Model formulations

We split the population into four groups: the susceptible group (S) in which all individuals admit normal eating behaviors and can be affected to become overeating, the group (I) that every individual exhibits the overeating behaviors, the group (T) in which individuals are treated to enter into the group (R) that all individuals are immune to overeating behaviors. The members in the group S and group I are further divided into two categories according to body mass index (BMI): the category of individuals with the normal BMI (BMI <25 kg/m^2) and the category of overweight members with the higher BMI (BMI≥ 25 kg/m^2) [21]. Let $S_N(t)$ and $S_O(t)$ be the numbers of individuals in group S with normal weights and higher weights at time t respectively, $I_N(t)$ and $I_O(t)$ be the numbers of individuals in group I with normal weights and higher weights at time t respectively, $T(t)$ and $R(t)$ be the numbers of individuals in the group T and group R at time t respectively. Let $S(t)=S_N(t)+S_O(t)$ and $I(t)=I_N(t)+I_O(t)$ denote the total sizes of susceptible group, overeating group at time t respectively.

Previous studies [4] indicate that overeating behaviors are driven by social pressures from friends, siblings, spouses, and neighbors. As a result, it is reasonable to assume that the transition rates of individuals in category S_N and category S_O to overeating group are described respectively by

$$\beta_N(\alpha I_N+I_O)S_N,\ \beta_O(\alpha I_N+I_O)S_O,$$

where β_N and β_O are the transmission coefficients of overeating individuals to susceptible individuals with the normal BMI and the higher BMI respectively, $0\leq\alpha\leq 1$ is a weight coefficient that represents the relative intensity of overeating behaviors with normal BMI to susceptible individuals.

Let μ be the recruitment rate of the population with the fraction p to the category S_N and the fraction $1-p$ to the category S_O. The fraction p is attributed to genetic factors. We assume that only individuals with the higher BMI are treated. Let ξ be the transition rate by which individuals enter into the compartment in treatment, γ be the recovery rate of individuals due to treatment and δ be the relapse rate of treated individuals. For the simplicity of notation, we assume that the removing rate of individuals in all compartments is the same, which is denoted by μ. Motivated by [2,5], we assume that there