Ann. of Appl. Math.
35:2(2019), 159-179

THE CAUCHY PROBLEMS FOR DISSIPATIVE HYPERBOLIC MEAN CURVATURE FLOW*†

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Abstract

In this paper, we investigate initial value problems for hyperbolic mean curvature flow with a dissipative term. By means of support functions of a convex curve, a hyperbolic Monge-Ampère equation is derived, and this equation could be reduced to the first order quasilinear systems in Riemann invariants. Using the theory of the local solutions of Cauchy problems for quasilinear hyperbolic systems, we discuss lower bounds on life-span of classical solutions to Cauchy problems for dissipative hyperbolic mean curvature flow.

Keywords dissipative hyperbolic mean curvature flow; hyperbolic Monge-Ampère equation; lifespan

2000 Mathematics Subject Classification 58J45; 58J47; 53C44; 35L70; 35L45

1 Introduction

In this paper, we study the closed convex evolving plane curves under the dissipative hyperbolic mean curvature flow. More precisely, we consider such an initial value problem

$$\begin{cases} \dfrac{\partial^2\gamma}{\partial t^2}(u,t)=\Big(k(u,t)-\Big(\alpha\dfrac{\partial\gamma}{\partial t},\vec{N}\Big)\Big)\vec{N}(u,t)-\nabla\rho(u,t), \quad \alpha\geq 0, \text{ for any } (u,t)\in S^1\times[0,T), \\ \gamma(u,0)=\gamma_0(u), \\ \dfrac{\partial\gamma}{\partial t}(u,0)=f(u)\vec{N}_0, \end{cases} \tag{1.1}$$

*The author was partially supported by Shandong Provincial Natural Science Foundation (Grant ZR2015AL008), the National Science Foundation for Young Scientists of China (Grant No.11001115, No.11201473), and the PHD Foundation of Liaocheng University (31805).

†Manuscript received July 25, 2018; Revised April 24, 2019

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where $k, \vec{N}, \gamma_0, f(u)(\geq 0)$ and $\vec{N}_0$ are the mean curvature, the unit inner normal, a smooth strictly convex closed curve, the initial velocity and the initial unit inner normal vector, respectively. $\vec{T}$ denotes the unit tangent vector, and s is the arclength parameter. $\nabla\rho$ is defined by

$$\nabla\rho = \left(\frac{\partial^2\gamma}{\partial s\partial t}, \frac{\partial\gamma}{\partial t}\right)\vec{T}.$$

This system is an initial value problem for a system of partial differential equations for γ, which can be completely reduced to an initial value problem for a single partial differential equation for its support functions. The latter equation is a hyperbolic Monge-Ampère equation. Next we present a local existence theorem for the initial value problems (1.1).

Theorem 1.1(Local existence and uniqueness) *Suppose that γ_0 is a smooth strictly convex closed curve. Then there exist a positive T and a family of strictly convex closed curves $\gamma(\cdot,t)$ with $t\in[0,T)$ such that $\gamma(\cdot,t)$ satisfies* (1.1), *provided that $f(u)$ is a smooth function on S^1.*

The following theorem has given the lifespan of classical solutions to the initial value problems.

Theorem 1.2 *Suppose that γ_0 is a strictly convex closed curve, and $f(u)$ is a smooth function on S^1, then the lower bound δ_* of the local solutions of dissipative hyperbolic mean curvature flow is*

$$\delta_* = \min\left\{\frac{\|\phi\|}{A_2}, \frac{A_0 + A_1\|\dot{\phi}\|}{B_0 + B_1\|\dot{\phi}\| + B_2\|\dot{\phi}\|^2}, \frac{1}{C_0 + C_1\|\dot{\phi}\|}\right\}, \tag{1.2}$$

where,

$$\phi = (\phi_1, \cdots, \phi_5)^{\mathrm{T}} = \left(\frac{1-\widetilde{f}'(\theta)}{g''(\theta)+g(\theta)}, \frac{-1-\widetilde{f}'(\theta)}{g''(\theta)+g(\theta)}, g(\theta), \widetilde{f}(\theta), g'(\theta)\right)^{\mathrm{T}},$$

$$\dot{\phi}_i(x) = \frac{\mathrm{d}\phi_i(x)}{\mathrm{d}x},$$

$$A_0(\|\phi\|) = 8\|\phi\|^3 + 4\|\phi\|^2 + (2\alpha+3)\|\phi\| + 1, A_1(\|\phi\|) = 2\|\phi\| + 1,$$

$$A_2(\|\phi\|) = A_0 - \|\phi\|, \quad A_3(\|\phi\|) = 48\|\phi\|^2 + 16\|\phi\| + 4\alpha + 4,$$

$$B_0(\|\phi\|) = 2A_0A_1A_3 + A_2, \quad B_1(\|\phi\|) = 6A_0A_1 + 2A_1^2A_3, \quad B_2(\|\phi\|) = 6A_1^2,$$

$$C_0(\|\phi\|) = 4(1+2\|\phi\|)^2(48\|\phi\|^2 + 16\|\phi\| + 4\alpha + 5)(12\|\phi\|^2 + 4\|\phi\| + \alpha + 1),$$

$$C_1(\|\phi\|) = 16(1+2\|\phi\|)^2(12\|\phi\|^2 + 4\|\phi\| + \alpha + 1).$$

It is well known that the elliptic and parabolic partial differential equations have been successfully applied to differential geometry and physics, such as Hamilton's Ricci flow and Schoen-Yau's solution to the positive mass conjecture [1,2]. It is na-