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Harmonic Standing-Wave Excitations of Simply-Supported Thick-Walled Hollow Elastic Circular Cylinders: Exact 3D Linear Elastodynamic Response

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Abstract. The forced-vibration response of a simply-supported isotropic thick-walled hollow elastic circular cylinder subjected to two-dimensional harmonic standing-wave excitations on its curved surfaces is studied within the framework of linear elastodynamics. Exact semi-analytical solutions for the steady-state displacement field of the cylinder are constructed using recently-published parametric solutions to the Navier-Lamé equation. Formal application of the standing-wave boundary conditions generates three parameter-dependent 6×6 linear systems, each of which can be numerically solved in order to determine the parametric response of the cylinder's displacement field under various conditions. The method of solution is direct and demonstrates a general approach that can be applied to solve many other elastodynamic forced-response problems involving isotropic elastic cylinders. As an application, and considering several examples, the obtained solution is used to compute the steady-state frequency response in a few specific low-order excitation cases. In each case, the solution generates a series of resonances that are in exact correspondence with a unique subset of the natural frequencies of the simply-supported cylinder. The considered problem is of general theoretical interest in structural mechanics and acoustics and more practically serves as a benchmark forced-vibration problem involving a thick-walled hollow elastic cylinder.

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1 Introduction

The vibration of an isotropic thick-walled hollow elastic circular cylinder is one of the classical applied problems of elastodynamics and has been of longstanding general interest to applied mathematicians, acousticians, engineers, and physicists [1–4]. There

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is a vast literature on the free vibration of finite-length isotropic hollow elastic circular cylinders, and while much of the fundamental work on the subject was carried out prior to the 1980s, there has, over the last three decades, been a steady stream of publications devoted to developing and testing various methodological approaches to obtaining natural frequencies and mode shapes under a variety of end conditions (see, for example, [5–14] and references therein). The literature on the forced vibration of hollow elastic cylinders (finite-length, isotropic, circular, or otherwise) is much smaller in comparison. Forced-vibration analyses of thick-walled elastic cylinders based on the exact three-dimensional (3D) theory of linear elasticity, in particular, remain scarce. In the context of forced-vibration analyses based on the Navier-Lamé equation of motion, the most notable and general work is due to Ebenezer and co-workers [15] (ERP), who devised an exact series method to determine the steady-state vibration response of a finite-length isotropic hollow elastic circular cylinder subjected to arbitrary axisymmetric excitations on its surfaces.

As pointed out in many reviews (see, for example, [3]), many fundamental forced-vibration problems involving hollow elastic cylinders have not yet been studied or solved using the exact 3D theory of linear elasticity. A useful and analytically tractable model problem that has been surprisingly overlooked is the steady-state vibration response problem for a simply-supported isotropic thick-walled hollow elastic circular cylinder subjected to arbitrary excitations on its curved surfaces. For a simply-supported cylinder, arbitrary asymmetric excitations on the curved surfaces can be naturally expressed as superpositions of two-dimensional (2D) harmonic standing waves in the circumferential and axial directions. Thus, the precursor is to consider individual 2D harmonic standing-wave excitations on the curved surfaces. This latter problem is theoretically significant in its own right since it is one of the few model problems for which the effects of individual harmonic excitations on the curved surfaces of the cylinder can be isolated and studied without having to incorporate non-trivial corrections in order to simultaneously satisfy the end conditions. In general, the exact nature of these effects will be obscured by other excitations needed to generate the desired end conditions.†

The free-vibration analog of the proposed problem, that is, the free-vibration problem for a simply-supported isotropic (thick-walled) hollow elastic circular cylinder, is an important benchmark problem in many numerical free-vibration studies (see, for example, [13] and references therein). Explicit analytical formulations and mathematical analyses of this problem are however not easy to find in the literature. Weingarten and Reismann [16] applied the method of eigenfunction expansions to this problem and obtained an implicit solution in 1974. The free-vibration analog can also be extracted as a special case of the largely overlooked work of Prasad and Jain [17], who, already in the mid 1960s, considered the problem of free harmonic standing waves in a simply-

†Incidentally, a corollary of ERP's work [15] is that arbitrary axisymmetric excitations on the curved surfaces of a finite-length hollow elastic cylinder can be mathematically expressed as (finite or infinite) superpositions of one-dimensional harmonic standing waves in the axial direction. Curiously, the effect of the individual harmonics on the steady-state vibration response was not considered in [15].