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EULER APPROXIMATION FOR NON-AUTONOMOUS MIXED STOCHASTIC DIFFERENTIAL EQUATIONS IN BESOV NORM*†

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Abstract

We consider a kind of non-autonomous mixed stochastic differential equations driven by standard Brownian motions and fractional Brownian motions with Hurst index $H \in (1/2, 1)$. In the sense of stochastic Besov norm with index γ, we prove that the rate of convergence for Euler approximation is $O(\delta^{2H-2\gamma})$, here δ is the mesh of the partition of $[0, T]$.

Keywords Brownian motion; fractional Brownian motion; Euler approximation; rate of convergence; Besov norm

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1 Introduction

Consider a mixed stochastic differential equation (SDE) involving standard Brownian motion and fractional Brownian motion (fBm) with Hurst index $H \in (1/2, 1)$, both defined on the stochastic basis $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0 \le t \le T}, \mathcal{P})$, for $t \in [0, T]$,

$$X_t = X_0 + \int_0^t a(s, X_s)\mathrm{d}s + \int_0^t b(s, X_s)\mathrm{d}W_s + \int_0^t c(s, X_s)\mathrm{d}B_s^H, \tag{1}$$

where X_0 is an $\mathcal{F}_0$-measurable random variable, $\mathbb{E}|X_0|^2 < \infty$, the stochastic integral with respect to standard Brownian motion $\{W_t\}_{t \in [0,T]}$ and fBm $\{B_t^H\}_{t \in [0,T]}$ are interpreted as Itô and pathwise Riemann-Stieltjes integral respectively. Assumptions on the coefficients will be given in Section 3.

Since the seminal paper [2], mixed stochastic models containing both a standard

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Brownian motion and an fBm gained a lot of attention, see [3,4,8,10,11,15,16], and so on. They allow us to model systems driven by a combination of random noises, one of which is white and another has a long memory. The motivation to consider such equations comes from some financial applications, where Brownian motion as a model is inappropriate because of the lack of memory, and fBm with $H > 1/2$ is too smooth. A model driven by both processes is free of such drawbacks.

Recently, there are a lot of works devoted to Euler approximation of mixed SDEs driven by standard Brownian motions and fBms, e.g. [3,6,8,9]. Guerra and Nualart [3] proved an existence and uniqueness theorem for weak solution of SDE (1) on $\mathbb{R}^d$. The proof relies on an estimate of Euler approximation for SDE (1) which was obtained using the methods of fractional integration and the classical Itô stochastic calculus. Mishura and Shevchenko [8] obtained that SDE (1) has a uniqueness strong solution by Euler approximation under some mild regularity assumptions and the space with the norm

$$\|X\|_\gamma := \sup_{t\in[0,T]} \left(|X_t| + \int_0^t \frac{|X_t - X_s|}{(t-s)^{1+\gamma}} \right) < \infty, \tag{2}$$

for some $\gamma \in (1-H, 1/2)$. In the same year they [9] also considered the following mixed SDE involving both standard Brownian motion and fractional Brownian motion with Hurst index $H > 1/2$, for $t \in [0,T]$,

$$X_t = X_0 + \int_0^t a(s, X_s)\mathrm{d}s + \int_0^t b(s, X_s)\mathrm{d}W_s + \int_0^t c(X_s)\mathrm{d}B_s^H. \tag{3}$$

Under the bound of $a(t,x), b(t,x), c(x)$ ($c(x) > 0$) together with their derivatives, and $(2H-1)$-Hölder continuous in time t, they showed that the mean-square rate of convergence of Euler approximation of solution to SDE (3) is $O(\delta^{\frac{1}{2}\wedge(2H-1)})$. Very recently under the same conditions Liu and Luo [6] got the better mean-square rate of convergence $O(\delta^{\frac{1}{2}})$ by a modified Euler method for SDE (3).

The aim of this paper is to prove that the rate of convergence for Euler approximation of SDE (1) is $O(\delta^{2H-2\gamma})$ in Besov norm (see Definition 3.1. To the best of our knowledge, up to now, there is no paper investigating the convergent rate for Euler approximation of SDE (1) in Besov norm. Thus, we will make a first attempt to research such problem in the present paper.

The rest of this paper is organized as follows. Some elements of fractional calculus on an interval are give in Section 2. The assumptions are presented and the existence and uniqueness result is shown in Section 3. Then, we devote to Euler approximation for SDE (1) and give the rate of convergence in Section 4. Finally, in Section 6, we make a conclusion of our work.