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BICYCLIC GRAPHS WITH UNICYCLIC OR BICYCLIC INVERSES*†

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Abstract

A graph G is nonsingular if its adjacency matrix $A(G)$ is nonsingular. A nonsingular graph G is said to have an inverse G^+ if $A(G)^{-1}$ is signature similar to a nonnegative matrix. Let $\mathcal{H}$ be the class of connected bipartite graphs with unique perfect matchings. We present a characterization of bicyclic graphs in $\mathcal{H}$ which possess unicyclic or bicyclic inverses.

Keywords inverse graph; unicyclic graph; bicyclic graph; perfect matching

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1 Introduction

Let G be a simple, undirected graph on n vertices. We denote its vertex set by $V(G)$ and its edge set by $E(G)$. We use P_n to denote the path on n vertices. And we use $[i,j]$ to denote an edge between the vertices i and j. The adjacency matrix $A(G)$ of G is a square symmetric matrix of size n whose (i,j)th entry a_{ij} is 1 if $[i,j]\in E(G)$ and 0 otherwise.

A graph G is nonsingular if its adjacency matrix $A(G)$ is nonsingular. Let G be an unweighted graph and G_W be the positive weighted graph obtained from G by giving weights to its edges using the positive weight function $W\colon E(G)\to(0,\infty)$.

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The unweighted graph G may be viewed as a weighted graph where each edge has weight 1. A perfect matching in a graph G is a collection of vertex disjoint edges that covers every vertex. If a graph G has a unique perfect matching, then we denote it by $\mathcal{M}$. In addition, when u is a vertex, we shall always use u' to denote the matching mate for u, where the edge $[u, u'] \in \mathcal{M}$. If G is a bipartite graph with a unique perfect matching then it is nonsingular (see [2]).

A unicyclic graph G is a connected simple graph which satisfies $|E(G)| = |V(G)|$. A bicyclic graph G is a connected simple graph which satisfies $|E(G)| = |V(G)| + 1$. There are two type of basic bicyclic graphs: ∞-graphs and θ-graphs. More concisely, an ∞-graph, denoted by $\infty(p, q, l)$, is obtained from two vertex-disjoint cycles C_p and C_q by connected one vertex of C_p and one of C_q with a path P_l of length $l - 1$ (in the case of $l = 1$, identifying the above two vertices); and a θ-graph, denoted by $\theta(p, q, l)$, is a union of three internally disjoint paths P_{p+1}, P_{q+1}, P_{l+1} of length p, q, l respectively with common end vertices, where $p, q, l \geq 1$ and at most one of them is 1. Observe that any bicyclic graph G is obtained from an ∞-graph or a θ-graph by attaching trees to some of its vertices (see [15]).

One motivation for considering a connected bipartite graph with a unique perfect matching is that in some cases in quantum chemistry, an Hückel graph can be considered as a connected bipartite graph with a unique perfect matching.

We say λ is an eigenvalue of G if λ is an eigenvalue of $A(G)$. We use $\sigma(G)$ to denote the spectrum of G.

In 1976, the notion of an inverse graph was introduced by Harary and Minc (see [5]). A nonsingular graph G is invertible if $A(G)^{-1}$ is a matrix with entries from $\{0, 1\}$, and the graph H with adjacency matrix $A(G)^{-1}$ is called the inverse graph of G. However, in the same article, when only one connected graph is invertible, the author proved that a connected graph G is invertible if and only if $G = P_2$. In 1985, another notion of an inverse graph was supplied by Godsil (see [2]). This concept generalizes the definition given by Harary and Minc.

Let $\mathcal{H}$ denote the class of connected bipartite graphs with unique perfect matchings. Let $G \in \mathcal{H}$, then $A(G)^{-1}$ is signature similar to a nonnegative matrix, that is, there exists a diagonal matrix S with diagonal entries from $\{1, -1\}$ such that $SA(G)^{-1}S \geq 0$. The weighted graph associated to the matrix $SA(G)^{-1}S \geq 0$ is called the inverse of G and is denoted by G^+. A invertible graph G is said to be a self-inverse graph if G is isomorphic to its inverse graph. Let $\mathcal{H}_g$ denote the class of connected bipartite graph with unique perfect matching $\mathcal{M}$ such that $G/\mathcal{M}$ is bipartite.

Definition 1[7] Let $G \in \mathcal{H}$, then G has at least two pendant (degree one) vertices. An edge of a graph is said to be pendant if one of its vertices is a pendant vertex.