COMMUNICATIONS IN MATHEMATICAL RESEARCH
32(2)(2016), 142–150

Fixed Point Theorems of the Iterated Function Systems

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Abstract: In this paper, we present some fixed point theorems of iterated function systems consisting of α-ψ-contractive type mappings in Fractal space constituted by the compact subset of metric space and iterated function systems consisting of Banach contractive mappings in Fractal space constituted by the compact subset of generalized metric space, which is also extensively applied in topological dynamic system.

Key words: fixed point, α-ψ-contractive mapping, iterated function system, generalized metric space

2010 MR subject classification: 47H10, 54H25

Document code: A

Article ID: 1674-5647(2016)02-0142-09

DOI: 10.13447/j.1674-5647.2016.02.07

1 Introduction and Preparatory Results

The most well known result in the theory of fixed points is Banach's contraction mapping principle. Samet *et al.*[1] proved the theorem of existence of fixed point of an α-ψ-contractive mapping in complete metric space. They discussed the Banach contraction principle with some generalized contraction conditions and weakened the usual contraction condition.

A lot of generalizations of metric spaces exist. Most of them were introduced in an attempt to extend some fixed point theorems known from the metric case (see [2]–[4]). Hence a generalized metric space (g.m.s) has been defined as a metric space in which the triangle inequality is replaced by the quadrilateral inequality. Many researches have studied the fixed point theorem on the complete metric space $(X,\ d)$, however, there are few results for the existence of fixed point on the complete metric space $(H(X),\ h)$ with the use of fixed point theorem on $(X,\ d)$ (see [5]–[9]). In general, even though f is ψ-contraction on $(X,\ d)$, it cannot be concluded that F_f is ψ-contraction on $(H(X),\ h)$.

The aim of this paper is to obtain the fixed point theorems of the some generalized

Received date: April 24, 2015.
Foundation item: The NSF (11271150) of China and China Government Scholarship.
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contractions in metric space $(H(X),\ h)$ and present the fixed point theorems of Banach contractions in metric space $(H(X)^*,\ h^*)$.

Before we establish the fixed point theorems in metric space $(H(X),\ h)$, we discuss some basic results. Samet *et al.*[1] proved the following results in the complete metric space.

Denote by Ψ the family of non-decreasing functions $\psi : [0,+\infty) \to [0,+\infty)$ such that $\sum\limits_{n=1}^{+\infty} \psi^n(t) < +\infty$ for each $t > 0$, where ψ^n is the n-th iteration of ψ.

Definition 1.1 ([1], Def. 2.1) *Let $(X,\ d)$ be a metric space and $f : X \to X$ be a given mapping. We say that f is an α-ψ-contractive mapping if there exist two functions $\alpha : X \times X \to [0,+\infty)$ and $\psi \in \Psi$ such that*

$$\alpha(x,y)d(f(x),f(y)) \leq \psi(d(x,y))$$

for all $x, y \in X$.

If $\alpha(x,y) = 1$ for all $x, y \in X$ and $\psi(t) = kt$ for all $t \geq 0$ and some $k \in [0,1)$, then $f : X \to X$ satisfies the Banach contraction principle. There is an example involving a function f that is not continuous (see [1], Example 2.4).

If $\alpha(x,y) = 1$ for all $x, y \in X$ and $\lim\limits_{n\to+\infty} \psi^n(t) = 0$ for all $t > 0$ (not necessarily $\sum\limits_{n=1}^{+\infty} \psi^n(t) < +\infty$ for all $t > 0$), then $f : X \to X$ satisfies a condition of the Matkowski's contraction theorem (see [10], Th. 1.6).

Definition 1.2 ([1], Def. 2.2) *Let $f : X \to X$ and $\alpha : X \times X \to [0,+\infty)$. We say that f is α-admissible if for all $x, y \in X$,*

$$\alpha(x,y) \geq 1 \Rightarrow \alpha(f(x),\ f(y)) \geq 1.$$

Theorem 1.1 ([1], Th. 2.1) *Let $(X,\ d)$ be a complete metric space and $f : X \to X$ be an $\alpha - \psi$-contractive mapping satisfying the following conditions*:

(1) *f is α-admissible*;

(2) *There is an $x_0 \in X$ such that $\alpha(x_0,\ f(x_0)) \geq 1$*;

(3) *f is continuous.*

Then f has a fixed point, *that is*, *there exists an $x^* \in X$ such that $f(x^*) = x^*$.*

To assure the uniqueness of the fixed point, we consider the following hypothesis:

(H) For all $x, y \in X$, there is a $z \in X$ such that $\alpha(x,z) \geq 1$ and $\alpha(y,z) \geq 1$.

Theorem 1.2 ([1], Th. 2.3) *Adding condition (H) to the hypotheses of Theorem* 1.1, *we obtain uniqueness of the fixed point of f.*

These fixed point theorems extended an earlier result of Banach's contraction principle and these results are a substantial generalization of Matkowski's fixed point theorem (see [5], [7]–[9]).