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ANALYSIS OF ROTHE METHOD FOR A VARIATIONAL -HEMIVARIATIONAL INEQUALITY IN ADHESIVE CONTACT PROBLEM FOR LOCKING MATERIALS

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Abstract. We study a system of differential variational–hemivariational inequality arising in the modelling of adhesive viscoelastic contact problems for locking materials. The system consists of a variational-hemivariational inequality for the displacement field and an ordinary differential equation for the adhesion field. The contact is described by the unilateral constraint and normal compliance contact condition in which adhesion is taken into account and the friction is modelled by the nonmonotone multivalued subdifferential condition with adhesion. The problem is governed by a linear viscoelastic operator, a nonconvex locally Lipschitz friction potential and the subdifferential of the indicator function of a convex set which describes the locking constraints. The existence and uniqueness of solution to the coupled system are proved. The proof is based on a time-discretization method, known as the Rothe method.

Key words. Variational-hemivariational inequality, Rothe method, adhesion, locking material, unilateral constraint, normal compliance, nonmonotone friction.

1. Introduction

In this paper, we discuss the solvability of a coupled system which consists of an abstract evolution variational-hemivariational inequality and an ordinary differential equation. The system serves as a model for numerous physics and engineering applications. We provide a theoretical illustration of our abstract results and study a quasi-stationary frictional contact problem with adhesion for viscoelastic locking materials. In this problem, the variational-hemivariational inequality describes the displacement field, the ordinary differential equation is for the adhesion field and the subdifferential of the indicator function of a convex set which describes the locking constraints.

Processes of adhesion are important in many industrial settings, such as parts, usually nonmetallic glued together and prevent delamination of composite materials. As a result, in order to obtain more precise models of contact phenomena, it is necessary to add adhesion to the description of contact problems. Here, we adopt the approach model of Frémond [10, 11] and introduce a surface internal variable, the bonding field, which takes values between zero and one, and which describes the fraction of active bonds on the contact surface. The number of literature on adhesive contact problem between a deformable body and a foundation grows rapidly, general models can be found in many contributions, such as [10, 11, 2, 5, 7, 9, 16, 25, 26].

For the locking materials, the strain tensor is constrained to stay in a given convex set. The study of elastic materials with locking effect was first introduced in the pioneering works of Prager [21, 22, 23]. There, the constitutive law of such materials was derived and different mechanical interpretations have been provided.

The main novelties of the paper are described as follows. First, we apply the Rothe method to study a system of a variational-hemivariational inequality and a differential equation. Until now, only few papers devoted to the Rothe method for

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variational-hemivariational inequalities, see [4, 3]. At the same time, they studied only a single variational-hemivariational inequality. The Rothe method to study a system of a hemivariational inequality and a differential equation was first studied in [19]. Here, we promote it to the system of a variational-hemivariational inequality and an ordinary differential equation.

Second, we study a new contact model for locking materials with short memory. Contact problems with locking materials have recently been considered in [1, 18, 27, 28]. For the problem considered in [18] the contact was described by the Signorini unilateral condition and the friction was modeled with a nonmonotone multivalued subdifferential condition. The existence and uniqueness to the problem were proved by using a surjectivity result for pseudomonotone operators as well as the Banach contraction principle. The reference [1] deals with the numerical analysis of the model considered in [18]. The reference [27] considered a model which was frictionless and described with a nonsmooth multivalued interface law which involves unilateral constraints and subdifferential conditions. The existence of a unique weak solution to the problem was proved, and its continuous dependence with respect to the bounds which govern the locking and the normal displacement was established.

We note that all models considered in the above mentioned papers were elliptic. And [28] deals with locking materials with long memory, this leads to a history-dependent inequality. In this paper, we deal with contact problem for locking materials with short memory, which leads to an evolutionary variational-hemivariational inequality.

Third, we show the existence of a unique weak solution to the contact model in this paper. Since the variational formulation of the contact problem consists of a variational-hemivariational inequality and an ordinary differential equation, it is a challenge to derive the existence and uniqueness of the solution for the coupled system.

The rest of the paper is structured as follows. In Section 2 we recall the notation and present some preliminary materials. In Section 3 we provide a classical and variational formulation of the adhesive contact model for locking materials. In section 4 we prove the main existence and uniqueness result, Theorem 3.1, and provide the proof by Rothe method.

2. Preliminaries

In this section, we recall some preliminaries which we will refer to in the sequel. We start with the definitions of Clarke directional derivative and Clarke subdifferential. Let X be a Banach space, X^* its dual. Denote by $\langle \cdot, \cdot \rangle_{X^* \times X}$ the duality pairing between X^* and X.

Definition 2.1. *Let $\psi\colon X \to \mathbb{R}$ be a locally Lipschitz function. The generalized directional derivative, in the sense of Clarke, of ψ at $x \in X$ in the direction $v \in X$, denoted by $\psi^0(x; v)$, is defined by*

$$\psi^0(x; v) = \limsup_{y \to x,\, \lambda \downarrow 0} \frac{\psi(y + \lambda v) - \psi(y)}{\lambda}$$

and the Clarke subdifferential of ψ at x, denoted by $\partial\psi(x)$, is a subset of a dual space X^* given by

$$\partial\psi(x) = \{\, \zeta \in X^* \mid \psi^0(x; v) \geq \langle \zeta, v \rangle_{X^* \times X} \ \forall\, v \in X \,\}.$$