

Dynamics Analysis of an SIS Epidemic Model with the Effects of Awareness*

Yanan Sun¹, Yakui Xue^{1,†}, Boli Xie¹ and Song Sun²

Abstract In this paper, an SIS model incorporating the effects of awareness spreading on epidemic is analyzed. Four kinds of equilibria of the model are given, and a new method is used to prove the stability of the equilibria. The threshold of awareness is R_1^a , which measures whether awareness spreads. When awareness does not spread, the basic reproduction number of disease is R_1^d , it is R_2^d when awareness spreads. The relationship among the three kinds of thresholds is discussed in details. Specially, the effects of various awareness parameters on epidemic are analyzed. Our theoretical results suggest that raising awareness can effectively reduce the basic reproduction number of disease and reduce the spread of disease. Furthermore, numerical simulations are performed to illustrate our results.

Keywords Awareness spreading, Epidemic model, Cooperative system, Stability.

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1. Introduction

In recent decades, many infectious diseases have broken out, including Ebola, swine flu, Zika virus and so on. At the end of 2019, an infectious disease named COVID-19 broke out in the world for the first time. It is highly infectious and no vaccine is available. In a short period of time, thousands of people were infected and many people died unfortunately. These epidemics seriously affect people's health and cause great economic losses. Due to the significant development of science and technology, the spread of awareness between people is becoming more and more convenient. Usually, the spread of disease information can raise people's awareness of the disease, which makes people take a series of protection measures [1]. When people try to prevent themselves from contracting an epidemic, the results may show lower susceptibility. Meanwhile, thanks to self-isolation or better sanitary conditions, the recovery time is shorter and the infectivity will also decrease [2]. Although the best way to control epidemic is to vaccinate, the cost of vaccination

[†]the corresponding author.

Email address: xyk5152@163.com(Y. Xue)

¹Department of Mathematics, North University of China, Taiyuan, Shanxi 030051, China

²Department of Civil Engineering, Hebei University of Architecture, Zhangjiakou, Hebei 075000, China

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is very high. Worse still, there are some diseases that do not have vaccines (such as malaria, Hepatitis C, etc.). In this case, raising the awareness of the epidemic can control the epidemic more effectively. Especially, in developing and underdeveloped countries, mass media campaigns can play an important role in changing public health-related behaviors [3]. In view of the complex changes in mass behavior under awareness circumstances, it is very important to understand how awareness affects the spread of epidemics.

Many mathematical models are proposed to study the effects of awareness on epidemics. These models can be divided into two major classes: network-based models [4 – 6] and mean-field models [7 – 9]. So far, there are two ways to explain the effects of awareness: (i) by introducing a mass media compartment to represent the public interaction with mass media [10 – 14], (ii) by changing the rate of diseases transmission and taking preventive measures [15, 16]. For the choice of the affecting way of awareness, we choose the case (ii) in this paper. For the case (ii), there have also been a few studies in recent years. Kiss et al. [1] extended a simple SIRS model to account for the treatment class. They proved that although the awareness of the whole population did not affect the epidemic threshold, but it could reduce the prevalence of infection. An SIRS model that considering the effects of private and public awareness on epidemic was studied [8, 17].

Medical research had shown that many epidemics, including some bacterial and sexually transmitted diseases, do not have permanent immunity. These epidemics can be constructed as SIS models [18, 19]. In this paper, we establish an SIS model to study the effects of awareness spreading on epidemic. There have been some related previous studies. Samanta et al. [20] used an SIS model to study the effects of awareness program in epidemics outbreak - a slow fast dynamics. Liu et al. [21] investigated the stochastic diseases dynamics of an SIS epidemic model on two patches incorporating media coverage. Granell et al. [22] established an SIS-UAU model, the dynamical interplay between awareness and epidemic spreading in multiplex networks was investigated.

Our research is based on [23], there are still many differences. More practical factors are considered than previous work. Due to the effects of awareness, the two types of basic reproduction numbers of disease are given. Further, the relationship among three thresholds and the effects of various parameters on the epidemic threshold are analyzed. These have not been analyzed in previous works. In short, a comprehensive and detailed theoretical analysis is provided in this paper, the content of our research is more in line with the real life. Therefore, the research of this paper carries certain theoretic meaning and applied value.

The organization of this paper is as follows. The SIS model is described in Section 2. The four kinds of equilibria and three kinds of threshold expressions are given in Section 3. The stability of equilibria is analyzed in Section 4. Parametric analysis and numerical simulation are in Section 5. The main conclusions are summarized in Section 6.

2. The model description

We divide the overall population into four compartments: susceptible and unaware (S^n), susceptible and aware (S^a), infected and unaware (I^n), infected and aware (I^a). Specially, in this paper, the unaware individuals are the individuals without disease awareness and the individuals who have disease awareness but do not take