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Global Stability of a Stochastic Lotka-Volterra Cooperative System with Two Feedback Controls*

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Abstract In this paper, a class of Lotka-Volterra cooperation system and corresponding stochastic system with two feedback controls which are affected by all species are considered. We obtain some sufficient criteria for local stability and global asymptotic stability of equilibria of the systems. Our study shows that these equilibria could be globally stable by adjusting coefficients of the feedback control variables and stochastic perturbation parameters. Numerical simulations are presented to demonstrate our main result.

Keywords Lotka-Volterra cooperative system, Feedback controls, Stochastic perturbation, Global stability.

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1. Introduction

The Lotka-Volterra system has been extensively investigated, see [1,2] and the references cited therein. During the last two decades, the study of dynamic behaviors of ecosystem with feedback controls has become one of important research topics, see [3,4] and the references cited therein. Some ecosystems have single feedback control strategy [5,6]. For example, [6] studied a cooperation system with one feedback control

$$\begin{cases} \frac{dx_1}{dt} = r_1 x_1(t)(\frac{K_1+\alpha_1 x_2(t)}{1+x_2(t)} - x_1(t) - d_1 u(t)), \\ \frac{dx_2}{dt} = r_2 x_2(t)(\frac{K_2+\alpha_2 x_1(t)}{1+x_1(t)} - x_2(t) - d_2 u(t)), \\ \frac{du}{dt} = -eu(t) + f_1 x_1(t) + f_2 x_2(t). \end{cases}$$

Another ecosystems have different feedback strategies to different species [4,7]. For instance, [7] considered a competitive system with two feedback controls

$$\begin{cases} \frac{dx_1}{dt} = x_1(t)(b_1 - a_{11}x_1(t) - a_{12}\int_0^{+\infty} K_1(s)x_2(t-s)ds - c_1 u_1(t)), \\ \frac{dx_2}{dt} = x_2(t)(b_2 - a_{21}\int_0^{+\infty} K_2(s)x_1(t-s)ds - a_{22}x_2(t) - c_2 u_2(t)), \\ \frac{du_1}{dt} = -e_1 u_1(t) + d_1 x_1(t), \\ \frac{du_2}{dt} = -e_2 u_2(t) + d_2 x_2(t). \end{cases}$$

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On the other hand, every feedback control variable in turn can be affected by one specie, such as the system in [7], or by more species, such as the system in [6]. These works showed that feedback control variables can usually influence the position of the positive equilibrium, and have no influence on the stability of the equilibrium under suitable conditions.

Meanwhile, the parameters involved in systems show random fluctuation due to environmental noise. Hence, many ecosystems with stochastic noise have been studied, see [8–12,14] and the references cited therein. For instance, [12] considered global stability of a stochastic SI epidemic model with feedback controls. They obtained that if the endemic equilibrium of the deterministic system is globally stable, then its corresponding stochastic system keeps the property provided the noise is sufficiently small.

Above phenomenons motivate us to propose and study the Lotka-Volterra type cooperative system with two feedback control variables as follows:

$$\begin{cases} dx_1(t) = x_1[b_1 - a_{11}x_1(t) + a_{12}x_2(t) - \alpha_1 u_1(t)]dt, \\ dx_2(t) = x_2[b_2 + a_{21}x_1(t) - a_{22}x_2(t) - \alpha_2 u_2(t)]dt, \\ du_1(t) = [-\eta_1 u_1(t) + a_1 x_1(t) + \beta_2 x_2(t)]dt, \\ du_2(t) = [-\eta_2 u_2(t) + \beta_1 x_1(t) + a_2 x_2(t)]dt, \end{cases} \tag{1.1}$$

where $a_{ij}, a_i, b_i, \eta_i, \alpha_i, \beta_i (i,j = 1,2)$ are positive constants. $x_i(t)(i = 1,2)$ is the density of population x_i at time t, and $u_i(t)(i = 1,2)$ is feedback control variable. As for as we know there is no global stability result on the cooperate Lotka-Volterra system with two feedback controls which are affected by every specie.

Moreover, we study a corresponding stochastic system

$$\begin{cases} dx_1(t) = x_1[b_1 - a_{11}x_1(t) + a_{12}x_2(t) - \alpha_1 u_1(t)]dt + \sigma_1 x_1(t)(x_1(t) - x_1^*)dB_1(t), \\ dx_2(t) = x_2[b_2 + a_{21}x_1(t) - a_{22}x_2(t) - \alpha_2 u_2(t)]dt + \sigma_2 x_2(t)(x_2(t) - x_2^*)dB_2(t), \\ du_1(t) = [-\eta_1 u_1(t) + a_1 x_1(t) + \beta_2 x_2(t)]dt, \\ du_2(t) = [-\eta_2 u_2(t) + \beta_1 x_1(t) + a_2 x_2(t)]dt, \end{cases} \tag{1.2}$$

where $B_i(t)(i = 1,2)$ is standard white noise, $\sigma_i(i = 1,2)$ denotes the intensity of the noise, $P(x_1^*, x_2^*, u_1^*, u_2^*)$ is unique positive equilibrium of (1.1), see (2.1) in Section 2. Obviously, when $\sigma_1 = \sigma_2 = 0$, system (1.2) will reduce to (1.1). And systems (1.1), (1.2) have same positive equilibrium. As far as we know there is no global stability result on the stochastic cooperate Lotka-Volterra systems with two feedback controls which are affected by all species. One interesting issue is whether the system still admits a unique globally stable positive equilibrium or the system could have more complicate dynamic behaviors under the influence of more complicated feedback controls and standard white noise?

The rest of the paper is organized as follows. In the next section, we investigate the local stability of the equilibria of system (1.1). Section 3 is assigned to discuss the global stability property of system (1.1) and stochastic system (1.2). In Section 4, examples together with their numerical simulations are presented to illustrate the feasibility of our main result. Finally, we end this paper by a briefly discussion.