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Exploring the Planar Circular Restricted Three-body Problem with Prolate Primaries

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Abstract We numerically investigate the convergence properties of the circular restricted three-body problem with prolate primaries, by using the Newton-Raphson iterative scheme. In particular, we examine how the oblateness coefficient A influences several aspects of the method, such as its speed and efficiency. Color-coded diagrams are used for revealing the basins of convergence on the configuration space. Additionally, we compute the degree of fractality of the convergence basins on the physical plane, as a function of the oblateness coefficient, by using different computational tools, such as the uncertainty dimension and the (boundary) basin entropy.

Keywords Restricted three-body problem, Oblateness parameter, Basins of convergence.

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1. Introduction

According to [1] in the original version of the restricted three-body problem (RTBP) the shape of the two primary bodies is assumed to be spherically symmetric. However, for obtaining a more realistic and complete representation regarding the nature of the motion of a body acting as a test particle, especially in the Solar System, a plethora of modifications have been proposed, over the years. All these modifications aim to include in the effective potential the influence of additional dynamical parameters, such as the shape or the radiation of the primary bodies.

It is well known that in our Solar System many celestial bodies (e.g., Saturn and Jupiter, as well many minor natural satellites) have a spheroidal shape [2]. For incorporating the particular shape of the primaries into the equations explaining the motion of the test body (e.g., comet, asteroid, or spacecraft) the oblateness parameter has been introduced and used initially in [3]. From then, a large amount of research work has been devoted on the study of the influence of the oblateness (see e.g., [4–16]).

As we know, in the original version of the RTBP there exist five equilibrium or Lagrange points. Unfortunately, there is the misbelief that the same number of points of equilibrium also exists in the case where the primary bodies are spheroidals, thus taking into account the oblateness coefficient. This however is correct only in the case of oblate primaries, where five equilibrium points are present for every positive value of the oblateness coefficient [17]. In the case of prolate primaries,

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that is when the parameter of the oblateness has negative values, the number of the equilibrium points is not constant, but a function of the oblateness. In the present paper we are going to present in detail the equilibrium points along with their linear stability, in the case of prolate primaries.

Knowing the coordinates of the points of equilibrium of a system is an issue of high importance. However, this is not possible for many complicated dynamical systems for which there are no analytical equations for the positions of the equilibrium points. This automatically means that only by using numerical methods we can obtain the locations of the libration points. As we know, in all numerical methods the initial conditions are very important. This is true because for some starting points the numerical methods may converge relatively fast to a root, while for other initial points they may require a considerable amount of iterations. Usually, points with fast convergence belong to the so-called basins of convergence (BOC), while points with slow convergence are situated in the vicinity of the fractal basin boundaries. Therefore, it is very important to know the location of the BOC for a dynamical system, because then we automatically are aware of the optimal initial conditions for the numerical methods. Here, we would like to point out that the BOC of a dynamical system strongly depend on the chosen numerical method. In other words, different numerical methods yield to completely different BOC, for the same dynamical system.

In Section 2 we provide the mathematical description of the dynamical model, while in the following section 3 we present the parametric evolution of the coordinates and the stability of the points of equilibrium. Section 4 we illustrate the geometry along with the basic properties of the Newton-Raphson BOC, while the Section 5 is devoted on determining the influence of the oblateness on the properties of the system of three bodies. In the final Section 6 we provide the discussion of our work.

2. Mathematical description of the dynamical system

The restricted three-body system (RTBP) contains two massive bodies P_1 and P_2 (known as the primaries), while the third body is assumed to act as a test particle [1]. This means that the mass m of the test particle is significantly smaller, comparing with masses m_1 and m_2 of the two primary bodies. The two main bodies rotate in circular orbits around their gravitational center (which is common), while the motion of the third body does have any dynamical impact on the motion of the primaries, due to its insignificant mass.

For convenience, we use a units system where the distance R, between the centers of the two main bodies, and the constant of gravity G are both equal to unity. Using the mass parameter $\mu = m_2/(m_1 + m_2) \leq 1/2$ we can express the dimensionless masses of the two main bodies as $m_1 = 1 - \mu$ and $m_2 = \mu$. Both centers of the primaries are located at $(x_1, 0, 0)$ and $(x_2, 0, 0)$ (on the x-axis), where of course $x_1 = -\mu$ and $x_2 = 1 - \mu$. In our analysis we use a rotating barycentric frame of reference $Oxyz$, where the Ox axis is the line containing the centers of the primaries, while the origin $(0, 0)$ of the system of coordinates coincides with the mass center of the primaries. Fig. 1 shows a schematic of the configuration of the system of bodies.