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N-dark Soliton Solutions for the Multi-component Maccari System*

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Abstract By virtue of the KP hierarchy reduction method, the N-dark soliton solutions of the multi-component Maccari system are constructed. Taking the coupled Maccari system for instance, the N-dark soliton solutions are further obtained in terms of determinants. In addition, in contrast with bright-bright soliton collisions, the dynamical analysis shows that the collisions of dark-dark solitons are elastic and there is no energy exchange among solitons in different components. What's more, we also investigate the dark-dark soliton bound states including stationary and moving ones.

Keywords N-dark soliton solutions, the multi-component Maccari system, KP hierarchy reduction method.

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1. Introduction

During the past decades, solitons in coupled nonlinear Schrödinger (CNLS) type equations have been studied intensively due to their intriguing interest and their applied realms. Many efforts have been made on coupled systems which describe the interaction of short wave packets with long waves in nonlinear dispersive media, as they are frequently used in the fields of nonlinear optics, fluid dynamics and others. Depending on the relative sign between the group velocity dispersion/diffraction and nonlinearity, solitons have two distinct types, namely, bright and dark solitons. The appearance of multi-component CNLS equations as dynamical equations in various areas of physics leads to the identification of bright-, dark-, bright-dark-, and dark-bright-type solitons. In the case of focusing Manakov system, it has been reported that there exhibit certain novel inelastic collision properties, which has not been observed in scalar counterpart. In addition, in the defocusing Manakov system, two bright-dark solitons can form a stationary bound state, in other words, solitons undergo elastic collision without shape change in this case. All these interesting interaction behaviors can be described by multi-soliton solutions in the underlying integrable system. To our knowledge, results are still scarce for the study on bright-dark and dark-dark types soliton propagation and their collision dynamics.

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Being motivated by the above reasons, in present work, we consider the Maccari system [16, 23, 26, 30]

$$i\Phi_t + \Phi_{xx} - u\Phi = 0, \tag{1.1}$$

$$u_y = (\sigma\Phi\Phi^*)_x, \tag{1.2}$$

where $\sigma = \pm 1, \Phi$ is complex while u is real; the asterisk represents complex conjugate hereafter. And this system was introduced by Maccari that usually used to describe the motion of localized isolated waves in physics. Its two-component generalization is given by

$$i\Phi_t^{(1)} + \Phi_{xx}^{(1)} - u\Phi^{(1)} = 0, \tag{1.3}$$

$$i\Phi_t^{(2)} + \Phi_{xx}^{(2)} - u\Phi^{(2)} = 0, \tag{1.4}$$

$$u_y = (\sigma_1\Phi^{(1)}\Phi^{(1)*} + \sigma_2\Phi^{(2)}\Phi^{(2)*})_x, \tag{1.5}$$

where $\sigma_1, \sigma_2 = \pm 1$.

For the Maccari system (1.1)-(1.2), many studies have been done. Uthayakumar etc investigated its integrability property by means of the singularity structure analysis [20]. In addition, Lai and Chow have obtained its two-dromion solutions based on the coalescence of wavenumbers technique [13]. By virtue of the variable separation approach, Zhang etc constructed many coherent soliton structures such as dromions, foldon and solitoff [27,28]. In recent work, its various rational solutions have been presented by He etc utilizing the Hirota's bilinear method [24]. And the general mixed N-soliton solutions of the multi-component Maccari system have been investigated by Han and Chen [9]. However, the general dark-dark N-soliton solutions of the multi-component Maccari system have not been exhibited so far.

It should be pointed out that the KP reduction technique is an effective way to derive soliton solutions of integrable systems, which was firstly investigated by the Kyoto school [11, 17]. So far it has been applied to construct soliton solutions in many equations such as the mKdV equation, the NLS equation and others. The reduction of Toda lattice hierarchy with constrained KP systems to derive dark and bright solitons has been established respectively [21, 22]. Additionally, by means of this reduction technique, Ohta and Yang have obtained the general N-dark-dark soliton solutions in the CNLS equations [18]. Also based on this method, the general bright-dark N-soliton solutions have already been found for the CNLS equations [7], the YO system [1] and the Mel'nikov system. Moreover, the KP reduction technique has also been applied to derive rational solutions including rogue waves solutions of integrable equations, see also the literatures [2, 14, 19, 25, 29].

The paper is organized. In Sec.2, by means of the KP reduction technique, we construct the general N-dark-dark soliton solutions of the two-component Maccari system. Besides, the dynamics of one and two solitons are discussed in Sec.3. In Sec.4, we derive the dark-dark soliton bound states, which contain the stationary and the moving ones. In Sec.5, the general N-dark-dark soliton solutions of the multi-component Maccari system in Gram determinant form are presented.