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Bifurcation of a Modified Leslie-Gower System with Discrete and Distributed Delays*

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Abstract A modified Leslie-Gower predator-prey system with discrete and distributed delays is introduced. By analyzing the associated characteristic equation, stability and local Hopf bifurcation of the model are studied. It is found that the positive equilibrium is asymptotically stable when τ is less than a critical value and unstable when τ is greater than this critical value and the system can also undergo Hopf bifurcation at the positive equilibrium when τ crosses this critical value. Furthermore, using the normal form theory and center manifold theorem, the formulae for determining the direction of periodic solutions bifurcating from positive equilibrium are derived. Some numerical simulations are also carried out to illustrate our results.

Keywords Modified Leslie-Gower system, discrete and distributed delays, stability, Hopf bifurcation.

MSC(2010) 34C23, 34K15.

1. Introduction

Recently, the dynamics (including stability, persistence, periodic oscillation, bifurcation and chaos, etc.) of predator-prey system has long been one of the dominant themes in mathematical ecology due to its universal existence and importance (for example, see [1, 7, 9, 11, 19, 26, 29]). [14] first proposed and discussed the following predator-prey system

$$\begin{cases} \dot{x}(t) = x(a - bx) - p(x)y, \\ \dot{y}(t) = y[s(1 - h\dfrac{y}{x})], \end{cases} \tag{1.1}$$

where $p(x)$ is the predator functional response to prey, a, s are the intrinsic growth rate of prey $x(t)$ and predator $y(t)$, respectively, b measures the strength of competition among individuals of species $x(t)$, $\frac{a}{b}$ is the environmental carrying capacity for the prey, the environmental carrying capacity of predator K_y is a function of the

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available prey quantity, where $K_y = \frac{x}{h}$. The predator subsistence exclusively dependent on prey population in system (1.1). However, in the case of severe scarcity of the prey x, the predator y can switch to other population, but its growth will be limited . By adding a positive constant to the K_y, that is, $K_y = \frac{x+c}{h}$, then in this case, a modified Leslie-Gower predator-prey system with functional response can be described by the following system

$$\begin{cases} \dot{x}(t) = x(a - bx) - p(x)y, \\ \dot{y}(t) = y[s(1 - h\dfrac{y}{x + c})]. \end{cases} \tag{1.2}$$

The system (1.2) have been studied by many authors(for example, see [3,4,25,30]). When $p(x)$ is Holling type II functional response, then system (1.2) can be written as

$$\begin{cases} \dot{x}(t) = x(t)(a_1 - bx(t) - \dfrac{c_1 y(t)}{x(t) + k_1}), \\ \dot{y}(t) = y(t)(a_2 - \dfrac{c_2 y(t)}{x(t) + k_2}). \end{cases} \tag{1.3}$$

For system (1.3), [2] investigated the boundedness of solution, existence of an attracting set and global stability of the coexisting interior equilibrium.

Time delays in mathematical models of population dynamics are usually due to gestation time, maturation time, capturing time or some other reasons. Therefore, a more realistic predator-prey system should be described by delay differential equations. In fact, delay differential equations are capable of generating rich effective and accurate dynamics compared to ordinary differential equations (for example, see [5,6,8,10,15–17,20,22–24,27,31]). [18] investigated the following delayed Leslie-Gower predator-prey system

$$\begin{cases} \dot{x}(t) = x(t)(a_1 - bx(t) - \dfrac{c_1 y(t)}{x(t) + k_1}), \\ \dot{y}(t) = y(t)(a_2 - \dfrac{c_2 y(t - \tau)}{x(t - \tau) + k_2}). \end{cases} \tag{1.4}$$

For system (1.4), Nindjin et al. investigated the permanence and global stability of positive equilibrium without considering the effects of time delays on the prey. [4] studied the following Leslie-Gower predator-prey system with delay and investigated Hopf bifurcations at the positive equilibrium

$$\begin{cases} \dot{x}(t) = x(t)(a_1 - bx(t) - \dfrac{c_1 y(t - \tau)}{x(t - \tau) + k_1}), \\ \dot{y}(t) = y(t)(a_2 - \dfrac{c_2 y(t - \tau)}{x(t - \tau) + k_2}). \end{cases} \tag{1.5}$$

Based on the above, in this paper, we will investigate the following modified Leslie-Gower predator-prey model with discrete and distributed time delays

$$\begin{cases} \dot{x}(t) = x(t)(a_1 - b\displaystyle\int_{-\infty}^{t} f(t - s)x(s)ds - \dfrac{c_1 y(t)}{x(t) + k_1}), \\ \dot{y}(t) = y(t)(a_2 - \dfrac{c_2 y(t - \tau)}{x(t - \tau) + k_2}), \end{cases} \tag{1.6}$$