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The Twin Domination Number of Strong Product of Digraphs

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Abstract: Let $\gamma^*(D)$ denote the twin domination number of digraph D and let $D_1 \otimes D_2$ denote the strong product of D_1 and D_2. In this paper, we obtain that the twin domination number of strong product of two directed cycles of length at least 2. Furthermore, we give a lower bound of the twin domination number of strong product of two digraphs, and prove that the twin domination number of strong product of the complete digraph and any digraph D equals the twin domination number of D.
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1 Introduction

Let $D = (V,\ A)$ be a finite digraph without loops and multiple arcs, where $V = V(D)$ is the vertex set and $A = A(D)$ is the arc set. For a vertex $v \in V(D)$, $N_D^+(v)$ and $N_D^-(v)$ denote the set of out-neighbors and in-neighbors of v, $d_D^+(v) = |N_D^+(v)|$ and $d_D^-(v) = |N_D^-(v)|$ denote the out-degree and in-degree of v in D, respectively. A digraph D is r-regular if $d_D^+(v) = d_D^-(v) = r$ for any vertices v in D. Given two vertices u and v in D, we say u out-dominates v if $u = v$ or $uv \in A(D)$, and we say v in-dominates u if $u = v$ or $uv \in A(D)$. Let $N_D^+[v] = N_D^+(v) \cup \{v\}$. A vertex v dominates all vertices in $N_D^+[v]$. A set $S \subseteq V(D)$ is a dominating set of D if S dominates $V(D)$. The domination number of D, denoted by $\gamma(D)$, is the minimum cardinality of a dominating set of D. The notion of twin domination in digraphs has been studied in [1]. A set $S \subseteq V(D)$ is a twin dominating set of D if each vertex of D is either in S or both, an out-neighbour of some vertex in S and an in-neighbour

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of some (possibly distinct) vertex in S. The twin domination number of D, denoted by $\gamma^*(D)$, is the minimum cardinality of a twin dominating set of D. Clearly, $\gamma(D) \leqslant \gamma^*(D)$.

Let $D_1 = (V_1,\ A_1)$ and $D_2 = (V_2,\ A_2)$ be two digraphs, which have disjoint vertex sets $V_1 = \{x_1, x_2, \cdots, x_{n_1}\}$ and $V_2 = \{y_1, y_2, \cdots, y_{n_2}\}$ and disjoint arc sets A_1 and A_2, respectively. The strong product $D_1 \otimes D_2$ has vertex set $V = V_1 \times V_2$ and $(x_i,\ y_j)(x_{i'},\ y_{j'}) \in A(D_1 \otimes D_2)$ if one of the following holds:

(a) $x_i x_{i'} \in A_1$ and $y_j y_{j'} \in A_2$;

(b) $x_i = x_{i'}$ and $y_j y_{j'} \in A_2$;

(c) $y_j = y_{j'}$ and $x_i x_{i'} \in A_1$.

The subdigraph $D_1^{y_i}$ of $D_1 \otimes D_2$ has vertex set

$$V_1^{y_i} = \{(x_j, y_i):\ \text{for any } x_j \in V_1,\ \text{fixed } y_i \in V_2\} \cong V_1,$$

and arc set

$$A_1^{y_i} = \{(x_j,\ y_i)(x_{j'},\ y_i):\ x_j x_{j'} \in A_1\} \cong A_1.$$

It is clear that $D_1^{y_i} \cong D_1$.

Similarly, the subdigraph $D_2^{x_i}$ of $D_1 \otimes D_2$ has vertex set

$$V_2^{x_i} = \{(x_i,\ y_j):\ \text{for any } y_j \in V_2,\ \text{fixed } x_i \in V_1\} \cong V_2,$$

and arc set

$$A_2^{x_i} = \{(x_i,\ y_j)(x_i,\ y_{j'}):\ y_j y_{j'} \in A_2\} \cong A_2.$$

It is clear that $D_2^{x_i} \cong D_2$.

In recent years, the domination number of the cartesian product of directed cycles and paths has been discussed in [2]–[7]. There are some related works for strong product of two digraphs (see, for example, [8]). In [9]–[15], it shows the recent related works of twin domination number of digraphs. However, to date no research about twin domination number has been done for strong product of directed cycles. In this paper, we study the twin domination number of $C_m \otimes C_n$. We mainly determine the exact values

$$\gamma^*(C_m \otimes C_n) = \begin{cases} \dfrac{mn}{4} & \text{if } m \text{ and } n \text{ are even;} \\ \dfrac{mn+n}{4} & \text{if } n \text{ is even and } m \text{ is odd;} \\ \left\lceil \dfrac{mn+n}{4} \right\rceil & \text{if } m \text{ and } n \text{ are odd.} \end{cases}$$

Furthermore, we give a lower bound of $\gamma^*(D_1 \otimes D_2)$ and prove that $\gamma^*(K_m \otimes D) = \gamma^*(D)$ for any digraph D.

2 Main Results

We denote the vertices of a directed cycle C_n by the integers $\{1, 2, \cdots, n\}$ considered modulo n. There is an arc xy from x to y in C_n if and only if $y = x + 1 \pmod{n}$. For any vertex $(i, j) \in V(C_m \otimes C_n)$,

$$N^+((i,\ j)) = \{(i,\ j+1),\ (i+1,\ j),\ (i+1,\ j+1)\},$$