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Equivalent Conditions of Complete Convergence for Weighted Sums of Sequences of Extended Negatively Dependent Random Variables

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Abstract: By using Rosenthal type moment inequality for extended negatively dependent random variables, we establish the equivalent conditions of complete convergence for weighted sums of sequences of extended negatively dependent random variables under more general conditions. These results complement and improve the corresponding results obtained by Li *et al.* (Li D L, RAO M B, Jiang T F, Wang X C. Complete convergence and almost sure convergence of weighted sums of random variables. *J. Theoret. Probab.*, 1995, **8**: 49–76) and Liang (Liang H Y. Complete convergence for weighted sums of negatively associated random variables. *Statist. Probab. Lett.*, 2000, **48**: 317–325).

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1 Introduction

In many stochastic model, the assumption that random variables are independent is not plausible. Many researchers focus on weakening the restriction of independence in recent years. The concept of extended negatively dependent random variables was firstly introduced

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by Liu[1] as follows.

Definition 1.1[1] *Random variables $\{X_i,\ i \geq 1\}$ are said to be extended negatively dependent if there exists a constant $M > 0$ such that both*

$$P\Big(\bigcap_{i=1}^{n}(X_i \leq x_i)\Big) \leq M \prod_{i=1}^{n} P(X_i \leq x_i) \tag{1.1}$$

and

$$P\Big(\bigcap_{i=1}^{n}(X_i > x_i)\Big) \leq M \prod_{i=1}^{n} P(X_i > x_i) \tag{1.2}$$

hold for each $n \geq 1$ and all real numbers $x_1,\ x_2,\ \cdots,\ x_n$.

In the case $M = 1$ the notion of extended negatively dependent random variables reduces to the well-known notion of so-called negatively dependent random variables which was introduced by Lehmann[2]. Recall that random variables $\{X_i, i \geq 1\}$ are said to be positively dependent if the inequalities (1.1) and (1.2) hold both in the reverse direction when $M = 1$. Not looking that the notion of extended negatively dependent random variables seems to be a straightforward generalization of the notion of negative dependence, the extended negative dependent structure is substantially more comprehensive. As it is mentioned in [1], the extended negatively dependent structure can reflect not only a negative dependent structure but also a positive one, to some extend. Joag-Dev and Proschan[3] also pointed out that negatively associated random variables must be negatively dependent, and therefore, negatively associated random variables are also extended negatively dependent. Some applications for sequences of extended negatively dependent random variables have been found. We refer to Shen[4] for the probability inequalities, Liu[1] for the precise large deviations, and Chen[5] for the strong law of large numbers and applications to risk theory and renewal theory.

The concept of complete convergence was firstly introduced by Hsu and Robbins[6] as follows. A sequence $\{X_n, n \geq 1\}$ of random variables is said to converge completely to a constant θ if $\sum\limits_{n=1}^{\infty} P(|X_n - \theta| > \epsilon) < \infty$ for any $\epsilon > 0$. In view of the Borel-Cantelli Lemma, the complete convergence implies almost sure convergence. Therefore the complete convergence is very important tool in establishing almost sure convergence. When $\{X_n, n \geq 1\}$ is a sequence of independent and identically distributed random variables, Baum and Katz[7] proved the following remarkable result concerning the convergence rate of the tail probabilities $P(|S_n| > \epsilon n^{1/p})$ for any $\epsilon > 0$, where $S_n = \sum\limits_{i=1}^{n} X_i$.

Theorem 1.1[7] *Let $\{X, X_n, n \geq 1\}$ be a sequence of independent and identically distributed random variables, $0 < p < 2$ and $r > 1$. Then*

$$\sum_{n=1}^{\infty} n^{r-2} P(|S_n| > \epsilon n^{1/p}) < \infty, \qquad \epsilon > 0,$$

if and only if

$$E|X|^{rp} < \infty,$$

where $EX = 0$ whenever $1 \leq p < 2$.