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Statistical Inference for the Parameter of Pareto Distribution Based on Progressively Type-I Interval Censored Sample

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Abstract: In this paper, the estimation of parameters based on a progressively type-I interval censored sample from a Pareto distribution is studied. Different methods of estimation are discussed, which include mid-point approximation estimator, the maximum likelihood estimator and moment estimator. The estimation procedures are discussed in details and compared via Monte Carlo simulations in terms of their biases.

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1 Introduction

A useful and tractable parametric model with relatively high probability in the upper tail is the Pareto distribution $f(y;\,\alpha,\,\theta)$ having probability density function

$$f(y;\,\alpha,\,\theta)=\alpha\theta^{\alpha}y^{-(\alpha+1)},\qquad 0\leq\theta<y, \tag{1.1}$$

and cumulative distribution function

$$F(y;\,\alpha,\,\theta)=1-\theta^{\alpha}y^{-\alpha},\qquad 0\leq\theta<y,$$

where $\alpha>0$ and $\theta>0$ are unknown parameters. Here we treat estimation of the shape parameter α that characterizes the tail, with the scale parameter θ assumed known. That is, we consider (1.1) as a single-parameter Pareto model, following [1–2]. In actuarial applications, (1.1) with θ known is appropriate when losses or claims below a certain level

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are not relevant (for example, when a deductible applies). In such a case, θ can represent the deductible, or sometimes a lesser value in order to incorporate inflation into the model, while ignoring data irrelevant to the issues under study. The parameter α, on which we focus, plays a key role in connection with determination of extreme quantiles, upper tail probabilities, mean excess functions, and excess-of-loss and stop-loss reinsurance premiums. For general overviews of the role of Pareto distribution and variants in actuarial science, econometrics, and other fields, see [3–4]. New application contexts continue to arise. For example, the cost distributions of combinatorial search algorithms have recently been shown to exhibit Pareto-type tail behavior (see [5]).

Inferences for the Pareto distribution were discussed by several authors. In case the data are complete, the maximum likelihood estimators (MLE) of α and θ are easy to calculate, and Quandt[6] proved that the MLE $\hat{\alpha}$ and $\hat{\theta}$ are consistent with the parameters α and θ, respectively. But it is complicated to compute MLE for censored data. Cohen and Whitten[7] proposed a moment estimator (ME) to estimate the parameters of Pareto distribution. Moreover, Geisser[8] has provided extensive analysis of the application of Bayesian methods in predicting the future values of this distribution of random variables from observed values in a complete random sample. Nigm and Hamdy[9] have also considered a similar problem under censored data. Pandey *et al.*[10] obtained the Bayesian estimator of the shape parameter using LINEX loss function. Wu[11] considered the maximum likelihood estimation problem based on progressive type-II censoring with random removals, and constructed the confidence intervals for the parameters and percentile of the lifetime distribution. Recently, Huang[12] has proposed an optimal estimation method for the shape parameter, probability density function and upper tail probability of the Pareto distribution based on a weighted empirical distribution function. In this article, we discuss the estimation of parameters based on a progressively type-I interval censored sample from the Pareto distribution.

In many life-testing and reliability studies, experimental units can be removed progressively from the experiment in different stages of the testing. The major reasons for removal of the experimental units are saving the working experimental units for future use, reducing the total time on test and lowering the cost. Data obtained from such experiments are called progressive censored data. Aggarwala[13] explored a union of type-I interval and progressive censoring and developed the statistical inference for exponential distribution for progressive type-I interval censored sample. Since then, statistical analysis for progressive type-I interval censored data has been studied by many authors (see [14–16]).

Progressive type-I interval censoring scheme can be described as follows. Consider n units which put on a test at time $t_0 = 0$. The units are inspected at m precified times $t_1, t_2, \cdots, t_m$, where t_m is the scheduled time to terminate the experiment. At the ith inspection time t_i, the number X_i of failures within $(t_{i-1}, t_i]$ is recorded and R_i surviving items are randomly removed from the life testing, for $i = 1, 2, \cdots, m$. As the numbers of surviving units at times $t_1, t_2, \cdots, t_m$, are random variables, the numbers of removal $R_1, \cdots, R_m$ can be determined as pre-specified percentages of the remaining surviving units. For