Communications
in
Mathematical
Research
30(2)(2014), 106–116

Approximation Theoretic Aspects of Probabilistic Representations for Bi-continuous C-semigroups

Cang Ding-bang[1], Chen Cang[1] and Song Xiao-qiu[2]

(*1. Department of Basic Curriculums, North China Institute of Science and Technology, Beijing, 101601*)

(*2. Collage of Science, China University of Mining and Technology, Xuzhou, Jiangsu, 221008*)

Communicated by Ji You-qing

Abstract: By means of Riemann-Stieltjes stochastic process, moment-generating functions and operator-valued mathematical expectation, the problem of probabilistic approximation for bi-continuous C-semigroups is studied and the general probabilistic approximation of exponential formulas and the generation theorems are given.

Key words: bi-continuous C-semigroup, probabilistic approximation, generation theorem

2010 MR subject classification: 47D05, 47D09

Document code: A

Article ID: 1674-5647(2014)02-0106-11

DOI: 10.13447/j.1674-5647.2014.02.02

1 Introduction

In 1940s, Hille[1] and Yosida[2] studied the theory of strongly continuous semigroups on Banach spaces. They were devoted to a qualitative and quantitative analysis for the solutions $u(t) = T(t)x$ of the initial value problem $u'(t) = Au(t)$ with $u(0) = x \in D(A)$, where A is the generator of the strong continuous semigroup and $D(A)$ is the domain of A. Since then on, the theory of strongly continuous semigroup has been matured and applied to different areas of science. However, it was clear that not every semigroup is strongly continuous and that a comprehensive theory requires a more general set-up. For this reason, many other classes of semigroups were studied, such as distribution semigroups, semigroups of growth

Received date: Feb. 6, 2012.

Foundation item: The NSF (10671205) of China, Fundamental Research Funds (3142012022, 3142013039 and 3142014039) for the Central Universities and the Key Discipline Construction Project (HKXJZD201402) of NCIST.

E-mail address: cdbjd@163.com (Cang D B).

of order α, integrated semigroups, convolution semigroups, and so on. It is then natural to look for suitable locally convex topologies weaker than the norm topology to treat the lack of strong continuity.

In order to treat one-parameter semigroups of linear operators on Banach spaces which are not strongly continuous, Kuhnemund[3] introduced a new class of semigroups called bi-continuous semigroups which are defined on a Banach space with an additional locally convex topology τ. This class of semigroups has two remarkable properties. There is variety of interesting semigroups belonging this class, such as Feller semigroups, Ornstein-Uhlenbeck semigroups, as well as certain evolutions semigroups. Albanese and Mangino[4] got some results on the convergence of bi-continuous semigroups, obtained a Lie-Trotter product formula, and applied it to Feller semigroups generated by second order elliptic differential operators with unbounded coefficients. Patricio[5] gave some introduction to bi-continuous semigroups and some remark on the Riemann-Stieltjes integral and discussed the Hille-Phillips functional calculus for generators of bi-continuous semigroups.

In the last decades, mathematicians have begun to use the powerful tool of probability theory to solve the semigroup approximation problem and achieved fruitful results, and related works can be found in [6–12]. The plan of this paper is as follows. In Sections 2 and 3, by means of Riemann-Stieltjes stochastic process, moment-generating functions, operator-valued mathematical expect and C bi-continuous modified modulus, the problem of probabilistic approximation for bi-continuous C-semigroups is studied, and the general probabilistic approximation of exponential formulas and the generating theorems are given.

2 Probabilistic Representations

We assume that the space X satisfies the following conditions. Let $(X,\tau)'$ be a Banach space with topological dual X', and τ a locally convex topology on X with the following properties:

(1) The space (X,τ) is sequentially complete on $\|\cdot\|$-bounded sets, i.e., every $\|\cdot\|$-bounded τ-Cauchy sequence converges in (X,τ);

(2) If the topology τ is coarser, then the $\|\cdot\|$-topology is a Hausdorff topology;

(3) The space $(X,\tau)'$ has a norm $(X,\|\cdot\|)$, i.e.,

$$\|x\| = \sup\{|\langle x,\phi\rangle| : \phi \in (X,\tau)',\ \|\phi\|_{(X,\|\cdot\|')} \le 1,\ x \in X\}.$$

Let P_τ denote a family of seminorms inducing from the locally convex topology τ on X and without loss of generality one can assume that $p(x) \le \|x\|$ for all $x \in X$ and $p \in P_\tau$. In this paper, all operators are linear and $D(A)$ denotes the domain of A. $NBV[0,\mathbf{R}]$ denotes the normalized functions of bounded variation in $[0,\mathbf{R}]$.

Definition 2.1[5] *Let X be a Banach space with a topology τ and $\alpha \in NBV[0,\mathbf{R}]$. A function $f:[0,\mathbf{R}]\to X$ is τ Riemann-Stieltjes integrable with respect to α if*

$$\int_0^R f(s)\mathrm{d}\alpha(s) : \tau - \lim_{\|\pi\|\to 0}\sum_{i=1}^{n}(\alpha(s_i)-\alpha(s_{i-1}))f(\xi_i)$$