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Principal Quasi-Baerness of Rings of Skew Generalized Power Series

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Abstract: Let R be a ring and $(S, \leq)$ be a strictly totally ordered monoid satisfying that $0 \leq s$ for all $s \in S$. It is shown that if λ is a weakly rigid homomorphism, then the skew generalized power series ring $[[R^{S,\leq}, \lambda]]$ is right p.q.-Baer if and only if R is right p.q.-Baer and any S-indexed subset of $\mathcal{S}_r(R)$ has a generalized join in $\mathcal{S}_r(R)$. Several known results follow as consequences of our results.

Key words: rings of skew generalized power series, right p.q.-Baer ring, weakly rigid endomorphism

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1 Introduction

Throughout this paper, R denotes an associative ring with unity. Recall that R is (quasi-) Baer if the right annihilator of every nonempty subset (every right ideal) of R is generated by an idempotent. In [1] Kaplansky introduced Baer rings to abstract various properties of AW^*-algebras and Von Neumann algebras. Clark defined quasi-Baer rings in [2] and used them to characterize when a finite dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. Further work on Baer and quasi-Baer rings appears in [3–6]. As a generalization of quasi-Baer rings, Birkenmeier, Kim and Park in [7] introduced the concept of principally quasi-Baer rings. A ring R is called right principally quasi-Baer (or simply right p.q.-Baer) if the right annihilator of every principal right ideal of R is generated by an idempotent. Similarly, left p.q.-Baer rings can be defined. A ring is called p.q.-Baer if it is both right and left p.q.-Baer. For more details and examples of right p.q.-Baer rings, see [8–13].

It was proved that a ring R is quasi-Baer if and only if $R[X]$ is quasi-Baer if and only

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if $R[[X]]$ is quasi-Baer, where X is an arbitrary nonempty set of not necessarily commuting indeterminates (see [3], Theorem 1.8). Birkenmeier *et al.*[8] showed that R is right p.q.-Baer if and only if $R[x]$ is right p.q.-Baer (see [8], Theorem 2.1), and an example ([8], Example 2.6) was given to show that the result is not true for $R[[x]]$. In [10], a necessary and sufficient condition was given for some rings under which the ring $R[[x]]$ is right p.q.-Baer (see [10], Theorem 3). It is shown that for a ring R with $\mathcal{S}_\ell(R) \subseteq C(R)$, $R[[x]]$ is right p.q.-Baer if and only if R is right p.q.-Baer and any countable family of idempotents in R has a generalized join in $I(R)$, where $I(R)$ is the set of all idempotents of R. In [11, 12], the author generalized the result to skew power series ring $R[[x;\ \alpha]]$ and generalized power series ring $[[R^{S,\leq}]]$. Cheng and Huang pointed out (see [9], Theorem 5) that the condition requiring all left semicentral idempotents being central is redundant in Theorem 3 of [10]. It is shown that $R[[x]]$ is right p.q.-Baer if and only if R is right p.q.-Baer and any countable subset of right semicentral idempotents has a generalized countable join. This properly generalized the result in [10], Theorem 3. Inspired by the results above, in this paper we investigate the principal quasi-Baerness of skew generalized power series rings. Let R be a ring, $(S,\ \leq)$ be a strictly totally ordered monoid such that $0 \leq s$ for all $s \in S$. It is shown that if λ is a weakly rigid homomorphism, then the skew generalized power series ring $[[R^{S,\leq},\ \lambda]]$ is right p.q.-Baer if and only if R is right p.q.-Baer and any S-indexed subset of $\mathcal{S}_r(R)$ has a generalized join in $\mathcal{S}_r(R)$. This generalize the results such as Theorem 5 in [11], Theorem 2.1 in [12], Theorem 5 in [9] and Theorem 4 in [13].

For a nonempty subset Y of R, $r_R(Y)$ denotes the right annihilator of Y in R. Let $C(R)$ be the set of all central elements of R.

Let $(S,\ \leq)$ be an ordered set. Recall that $(S,\ \leq)$ is artinian if every strictly decreasing sequence of elements of S is finite, and that $(S,\ \leq)$ is narrow if every subset of pairwise order-incomparable elements of S is finite. Let S be a commutative monoid. Unless stated otherwise, the operation of S is denoted additively, and the neutral element is denoted by 0. The following definition is due to [14, 15].

Let $(S,\ \leq)$ be a strictly ordered monoid (that is, $(S,\ \leq)$ is an ordered monoid satisfying the condition that, if $s, s', t \in S$ and $s < s'$, then $s + t < s' + t$), and R a ring. Let $[[R^{S,\leq}]]$ be the set of all maps $f : S \to R$ such that $\mathrm{supp}(f) = \{s \in S \mid f(s) \neq 0\}$ is artinian and narrow. With pointwise addition, $[[R^{S,\leq}]]$ is an abelian additive group. For every $s \in S$ and $f, g \in [[R^{S,\leq}]]$, let

$$X_s(f, g) = \{(u, v) \in S \times S \mid s = u + v,\ f(u) \neq 0,\ g(v) \neq 0\}.$$

It follows from [16] that $X_s(f, g)$ is finite. Denote by $\mathrm{End}(R)$ the set of all ring homomorphisms from R to R. Let $\lambda : S \to \mathrm{End}(R)$ be a map satisfying the following condition:

$$\lambda(u + v) = \lambda(u)\lambda(v), \qquad u, v \in S.$$

For any $s \in S$ and $f, g \in [[R^{S,\leq}]]$, define $fg : S \to R$ via

$$(fg)(s) = \sum_{(u,v)\in X_s(f,g)} f(u)\lambda(u)(g(v)).$$

With the addition and the multiplication as above, $([[R^{S,\leq}]],\ +,\ \cdot)$ becomes a ring, which we denote by $[[R^{S,\leq}, \lambda]]$, and call it the skew generalized power series ring related to λ. The