COMMUNICATIONS
IN
MATHEMATICAL
RESEARCH
28(1)(2012), 91–96

Two Generator Subsystems of Lie Triple System*

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Communicated by Du Xian-kun

Abstract: For a Lie triple system T over a field of characteristic zero, some sufficient conditions for T to be two-generated are proved. We also discuss to what extent the two-generated subsystems determine the structure of the system T. One of the main results is that T is solvable if and only if every two elements generates a solvable subsystem. In fact, we give an explicit two-generated law for the two-generated subsystems.

Key words: Lie triple system, two generated subsystem, solvable
2000 MR subject classification: 17A40, 17B05
Document code: A
Article ID: 1674-5647(2012)01-0091-06

1 Introduction

Lie triple system (L.t.s.) is a generalization of the concept of Lie algebra, since every Lie algebra L is also an L.t.s. with the multiplication

$$[x,\ y,\ z] := [x,\ [y,\ z]].$$

And also every L.t.s. is a subsystem of an L.t.s. coming from a Lie algebra, due to the concept of standard imbedding. Hence L.t.s. is strongly linked to Lie algebra, and many results of Lie algebra can be generalized in an appropriate form, to the L.t.s. (see [1–5]).

In this section we recall some definitions and facts about L.t.s. We start with the definition of an L.t.s. (see [1–2]).

Definition 1.1 *A Lie triple system (L.t.s.) is a vector space T over a field $\mathbb{K}$, which is closed with respect to a trilinear multiplication $[\ \cdot,\ \cdot,\ \cdot\ ]$ and satisfies*

$$[xxy] = 0, \tag{1.1}$$

$$[xyz] + [yzx] + [zxy] = 0, \tag{1.2}$$

$$[uv[xyz]] = [[uvx]yz] + [x[uvy]z] + [xy[uvz]], \tag{1.3}$$

*__Received date:__ July 14, 2010.
Foundation item: The NSF (A2007000138) of Hebei Provience.

where $u, v, x, y, z \in T$.

A derivation of an L.t.s. T is a linear transformation D of T such that

$$D[xyz] = [(Dx)yz] + [x(Dy)z] + [xy(Dz)], \qquad x, y, z \in T.$$

For $x, y, z \in T$, define linear transformations $L(\ \cdot,\ \cdot\ )$, $R(\ \cdot,\ \cdot\ )$ on the vector space T by

$$L(x, y)(z) = R(y, z)(x) = [xyz].$$

We can see by the definition of T that all $L(x, y)$, $x, y \in T$, are derivations. A derivation D of the form

$$D = \sum L(x_i,\ y_i), \qquad x_i, y_i \in T$$

is called an inner derivation.

A subspace U of T is called an ideal of T if for all $x, y \in T$, $u \in U$ we have $[uxy] \in U$. For any submodule V in T, the centralizer $Z_T(V)$ of V in T is defined by

$$Z_T(V) = \{x \in T \mid [xvt] = [tvx] = 0,\ t \in T,\ v \in V\}.$$

In particular, $Z_T(T)$ is called the center of T and denoted simply by $Z(T)$. An L.t.s. T is called abelian if it satisfies

$$[xyz] = 0, \qquad x, y, z \in T.$$

For an ideal V of an L.t.s. T, define the lower central series (see [6]) for V by

$$V^0 := V$$

and

$$V^{n+1} := [V^n T V] + [V T V^n], \qquad n \geq 0.$$

Then V is called T-nilpotent if $V^m = 0$ for some m. It is called nilpotent if it is T-nilpotent. Put

$$V^{(0)} := V, \qquad V^{(n+1)} := [V^{(n)} T V^{(n)}].$$

Then V is called solvable if there is a positive integer k for which $V^{(k)} = 0$. T is solvable if it is a solvable ideal.

The Frattini subsystem $F(T)$ of an L.t.s. T is the intersection of all maximal subsystems of T. The Frattini ideal $\phi(T)$ is the largest ideal of T contained in $F(T)$.

For a subset U of an L.t.s. T, $\langle U \rangle$ denotes the subsystem of T generated by U.

2 Two Generated Subsystems of L.t.s.

In this section we give some results about two generated subsystems of L.t.s., which are generalisations of corresponding results of Lie algebras (see [7]).

Definition 2.1 *Let T be an L.t.s. For arbitrary $x, y \in T$, the subsystem $\langle x,\ y \rangle$ generated by x, y is called a two-generated subsystem. T is called two-generated if it is a two-generated subsystem of itself.*

Lemma 2.1 *An L.t.s. T is two-generated if and only if $T/\phi(T)$ is two-generated.*